KINETIC THEORIES

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11 Kinetic Theories

Criticisms and accomplishments of the Prigogine-Herman kinetic theory are reviewed. Two of the latter are identified as possible benchmarks, against which to measure proposed novel kinetic theories of vehicular traffic. Various kinetic theories that have been proposed in order to eliminate deficiencies of the Prigogine-Herman theory are assessed in this light. None are found to have yet been shown to meet both of these benchmarks.

Possible objectives and applications for kinetic theories of vehicular traffic are considered. One of these is the traditional application to the development of continuum models, with the resulting microscopically based coefficients. However, modern computing power makes it possible to consider computational solution of kinetic equations *per se*, and therefore direct applications of the kinetic theory (e.g., the kinetic distribution function). It is concluded that the primary applications are likely to be found among situations in which variability between instances is an important consideration (e.g., travel times, or driving cycles).

11.1 Introduction

On page 20 of their well-known monograph on the kinetic theory of vehicular traffic, Prigogine and Herman (1971) summarize possible alternate forms of the relaxation term in their kinetic equation of vehicular traffic. They conclude this discussion by issuing the invitation that "the reader may, if he is so inclined, work out the theory using other forms of the relaxation law." This invitation to explore alternate kinetic models of vehicular traffic has subsequently been accepted by a number of workers, most notably by Paveri-Fontana (1975) and by Phillips (1979, 1977), and more recently by Nelson (1995a), and by Klar and Wegener (1999). The existence of this variety of kinetic models of vehicular traffic raises the issue of how one chooses between them in any particular application; more generally there arises the issue of the types of applications for which any kinetic model has a role. In these lights, the primary objective of this chapter is to address questions related to what might reasonably be expected from a good kinetic theory of vehicular traffic.

The approach presented here is substantially influenced by the work of Nagel (1996), who gave an excellent review of a variety of types of models of vehicular traffic, including continuum ("hydrodynamic"), car-following and particle hopping (cellular automata) models. In particular, he has emphasized that: *i)* any model necessarily represents some compromise in terms of its *fidelity* in describing the reality it is intended to represent; *ii)* different types of models represent engineering judgements as to the relative importance of *resolution*, fidelity and *scale* for the particular application at hand. To some extent, this chapter is intended to address similar issues for *kinetic* models of vehicular traffic. The status of various kinetic models will also be reviewed, in terms of achieving two objectives that seem appropriate to designate as benchmarks, primarily on the basis that the seminal kinetic model of Prigogine and Herman (1971) has been shown to meet those objectives.

It seems appropriate to view kinetic models as occupying a point on the model spectrum that is intermediate between continuum (e.g., hydrodynamic) models and microscopic (e.g., car-following or cellular automata) models.[1] One of the primary applications of kinetic models is to obtain continuum models in a consistent manner from an underlying microscopic model of driver behavior. (See Nelson (1995b) for further thoughts on the role of kinetic models of vehicular traffic as a bridge from microscopic models to macroscopic models.) However, computing power now has advanced to the point that it is practical to consider computational solution of kinetic equations *per se*. This opens the door to the realistic possibility of applying kinetic models directly to the simulation of traffic flow. This is a qualitatively different situation from that prevailing in the 1960's, when kinetic models of vehicular traffic were initially proposed by Prigogine, Herman and co-workers. (See Prigogine and Herman, 1971, and works cited therein.)

The specific further contents of this chapter are as follows. In Section 2 below, the status of the Prigogine-

[1] The word *mesoscopic* has come into recent vogue to describe models that are, in some sense, intermediate between macroscopic and microscopic models.

Herman (1971) kinetic model of vehicular traffic is reviewed. The intent of this review is to provide an evenhanded discussion of both the deficiencies and signal accomplishments of this seminal kinetic theory of vehicular traffic. Two of these accomplishments are suggested as benchmarks that should minimally be met by any proposed novel kinetic model of vehicular traffic. In Section 3 alternate kinetic models that have been proposed in the literature are assessed against these benchmarks, and none are found that yet have been shown to meet both of them. Both of these benchmarks relate to the equilibrium solutions of the Prigogine-Herman kinetic equation, and one of them relates to the recent result of Nelson and Sopasakis (1998) to the effect that under certain circumstances – particularly for sufficiently congested traffic – the Prigogine-Herman model admits a two-parameter family of equilibrium solutions, as opposed to the one-parameter (density) family that would be expected classically.

In Section 4, the role of kinetic equations as a bridge from microscopic to continuum models is considered. Section 5 is devoted to consideration of the potential applications of the solution of kinetic equations *per se.*

11.2 Status of the Prigogine-Herman Kinetic Model

The kinetic model of Prigogine and Herman (1971) is summarized in Subsection 2.1. A number of published criticisms of this model, along with alternative models that have been suggested to overcome some of these criticisms, are reviewed in Subsection 2.2. In Subsection 2.3 two significant accomplishments of the Prigogine-Herman theory are described, and suggested as benchmarks against which novel kinetic theories of vehicular traffic should be measured.

11.2.1 The Prigogine-Herman Model

The kinetic equation of Prigogine and Herman is

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = -\frac{f - f_0}{T} + c(\bar{v} - v)(1 - P)\,f. \qquad (11.\ 1)$$

Here the various symbols have the following meanings:

a) the zero order moment of *f(x,v,t),*

$$c(x,t) = \int_0^\infty f(x,v,t)\,dv,$$

is vehicular density.

b) the ratio of the first and zero order moments,

$$\bar{v}(x,t) = \int_0^\infty v f(x,v,t)\,dv \Big/ c(x,t)$$

is mean vehicular speed;

c) P is passing probability;

d) T is the relaxation time;

e) f_0 is the corresponding density function for the desired speed of vehicles;

f) f is the density function for the distribution of vehicles in phase space, so that

$$\int_{v_1}^{v_2}\int_{x_1}^{x_2} f(x,v,t)\,dv\,dx$$

is the expected number of vehicles at time t that have position between x_1 and x_2 and speed between v_1 and v_2 ($x_1 \le x_2$ and $v_1 \le v_2$).

The second term on the left-hand side of Eq. (1), the *streaming* term, represents the rate of change of the density function due to motion of the traffic stream, absent any changes of velocity by vehicles. The first term on the right-hand side, which is often called the *relaxation* term, is the contribution to this rate of change that stems from changes of vehicular speed associated with passing or other causes of acceleration. The second term on the right-hand side, the *slowing-down* term, stems from deceleration of vehicles that overtake slower-moving vehicles. The relaxation term is phenomenological in nature, in that it is based on the underlying assumption that increases in vehicular speed cause the actual density to "relax" toward the desired density with some characteristic time *T.* By contrast, the slowing-down term can be obtained from basic physical arguments, albeit with idealized assumptions such as instantaneous deceleration, treatment of vehicles as point particles (i.e., neglect of the positive length of vehicles), and the validity of what Paveri-Fontana (1975) terms *vehicular chaos.* The validity of both of these particular forms of the rates of change due to changes of speed has been questioned, as will be briefly discussed in the following subsection.

A *kinetic equation* generally is an equation that in principle, subject to appropriate initial and boundary conditions, can be solved for the density function *f*, as defined above. Some kinetic equations that are alternatives to that of Prigogine and Herman are discussed in Section 3 following

11.2.2 Criticisms of the Prigogine-Herman Model.

The first published serious critique of the Prigogine-Herman kinetic equation seems to be the work of Munjal and Pahl (1969). These workers raise a number of questions,[2] but the most fundamental of these fall into one of the following two categories:

1. The validity of the slowing-down term (denoted the "interaction term" by these authors) is doubtful in the presence of "queues" (or "platoons") of vehicles. This stems from the fact that the correlation inherent in platoons invalidates the assumption of vehicular chaos (Paveri-Fontana, 1975), which assumption underpins the particular form of the slowing-down term in the Prigogine-Herman kinetic equation.
2. The absence of a derivation of the relaxation term from first principles raises general questions regarding its validity. The validity of the specific expression (in terms of *c*) used by Prigogine et al. for the relaxation time *T* has therefore "not been proven." Further, it is therefore also difficult to "conceive the meaning of the relaxation time" and therefore "define a method for its experimental determination."

In addition to noting the first of these concerns, Paveri-Fontana (1975) argues forcefully that it is fundamentally incorrect to treat the desired speed as a parameter, as in done in the Prigogine-Herman kinetic equation. Rather, he suggests the desired speed must be taken as an additional independent variable, on the same footing as the actual speed, and he provides a modification of the Prigogine-Herman equation that accomplishes precisely that.

Prigogine and Herman (1971, Section 3.4, and 1970) dispute the claim of Munjal and Pahl (1969) to the effect that "the validity of the interaction term (i.e., the Prigogine-Herman slowing-down term) is limited to traffic situations where no vehicles are queuing" (parenthetical clarification added). Current opinion seems inclined to agree with Paveri-Fontana (1975) that on balance Munjal and Pahl have the better of this particular discussion. However, traffic on arterial roads, for which signalized intersections necessarily enforce the formation of platoons, is the only situation that seems thus to be definitely excluded from the domain of the Prigogine-Herman kinetic equation. In particular, it is not *a priori* clear that the same objection is valid for the stop-and-go traffic that seems to characterize congested traffic on freeways. Nelson (1995a) has noted that a *correlation model* is generally needed to obtain a kinetic equation, and vehicular chaos is simply one instance of a correlation model. Other correlation models, which would lead to a kinetic equation other than that of Prigogine and Herman, conceivably could better treat platoons, at least under restricted circumstances. Approaches (e.g., Prigogine and Andrews, 1960; Beylich, 1979), in which multiple-vehicle density functions appear as the unknowns to be determined, also offer the potential ability to treat queues within the spirit of the kinetic theory.

Nelson (1995a) introduced the concepts of a *mechanical model* and a *correlation model* as the fundamental ingredients of any kinetic equation. This work was motivated precisely by the desire to obtain forms of the speeding-up term that are based upon at least the same level of first principles as the classical derivations of the Prigogine-Herman slowing-down term. Klar and Wegener (1999) used this approach to obtain a kinetic equation for traffic flow that accounts for the spatial extent of vehicles. The treatment of vehicles as "points" of zero length is an idealization underlying the Prigogine-Herman kinetic equation that seems not to have been extensively discussed in the earlier literature on traffic flow. Klar and Wegener (1999) show that including the length of vehicles has a significant quantitative effect upon the value of some coefficients in associated continuum models. The observational measurements of the relaxation time by Edie, Herman and Lam (1980) also bear mentioning.

The arguments of Paveri-Fontana (1975) that the desired speed must appear as an independent variable in any kinetic equation, so that the density function depends upon the

[2] Other concerns relate to: *i)* The necessity to include time dependence in the desired speed distribution, owing to the normalization $\int_0^{v_{max}} f_0(x,v,t)\,dv = c(x,t)$; and *ii)* the interpretation and functional dependence of the passing probability, *P*.

desired speed, as well as position, actual speed and time, seem to be quite convincing. In order to avoid this complexity, some workers (e.g., Nelson, 1995a) choose to focus upon models in which all drivers have the same desired speed. Paveri-Fontana (1975) represents his modification of the Prigogine-Herman equation, to include desired speed as an independent variable, as valid only for dilute traffic. However, as suggested above, it is not completely clear that this restriction is required, unless the dense traffic also includes a significant fraction of the vehicles in platoons.

11.2.3 Accomplishments of the Prigogine-Herman Model

In view of the deficiencies chronicled in the preceding subsection, why would anyone deem the Prigogine-Herman kinetic equation to be of any interest? That question is answered in this subsection, by describing two significant results that stem from the Prigogine-Herman model.

First, Prigogine and Herman (1971, Chap. 4) demonstrated, under the somewhat restrictive assumption that there exist drivers desiring arbitrarily small speeds, that one can obtain traffic stream models (fundamental diagrams, speed/density relations), say $q=Q(c)$, from the equilibrium solutions (i.e., the solutions that are independent of space and time) of their kinetic equation. (Here q is vehicular flow, and c is, as above, vehicular density.) The procedure is precisely analogous to that giving rise to the Maxwellian distribution and the ideal gas law, when applied to the Boltzmann equation of the kinetic theory of gases. Further, at high concentrations the equilibrium solution is *bimodal*; that is, it displays two (local) maxima in speed, in qualitative agreement with the observations of Phillips (1977, 1979). (See the following section for more details of these works.) One of these modes corresponds to a modification of the distribution of desired speeds, and the other (under the assumptions of Prigogine and Herman) to platoon flow in the rather extreme case of stopped traffic (i.e., zero speed). This "multiphase" aspect of congested traffic flow has subsequently been rediscovered by a number of workers. Note that this approach gives rise to a traffic stream model from an underlying microscopic model, via the equilibrium solution of a corresponding kinetic equation. Such a theoretical development contrasts with statements sometimes encountered to the effect that traffic stream models *must* be based upon observational data.

More recently, Nelson and Sopasakis (1998) showed that if one relaxes the assumption of Prigogine and Herman that there exist drivers having arbitrarily small desired speeds, then at sufficiently high densities the equilibrium solution is a two-parameter family. This contrasts with the one-parameter (typically taken as density) family that occurs at low densities, even at all densities under the restrictive assumption of Prigogine and Herman (1971).[3]

The consequence of the equilibrium solutions of Nelson and Sopasakis (1998) for the attendant traffic stream model will now be briefly described. Let

$$F(\zeta;c) := \int_{w_-}^{w_+} \frac{f_0(v)}{c(v-\zeta)}\, dv,$$

where w_- and w_+ are respectively the lower and upper bound on the desired speeds. Then there exists a positive *critical density,* denoted c_{crit}, and defined as the unique root (in c) of the equation $F(0;c) = cT(1-P)$, such that the dependence of mean speed upon density is as follows. Let $\zeta^*=\zeta^*(c)$ be the unique root (in ζ) of $F(\zeta;c) = cT(1-P)$. If $0\le c\le c_{crit}$, then $\zeta^*\le 0$, and the mean speed is given by

$$\bar{v} = \bar{v}(c) = \frac{1}{cT(1-P)} + \zeta^* .$$

However, if $c>c_{crit}$, then $\zeta^*>0$, and the mean speed is given by

$$\bar{v} = \bar{v}(c,\zeta) = \frac{1}{cT(1-P)} + \zeta ,$$

where now ζ can take on any value such that $0 \le \zeta \le min\{\zeta^*, w_-\}$. The parameter ζ is the speed of the embedded collective flow, and the preceding equation for the mean speed shows that the overall mean speed increases with increasing speed of the embedded collective flow. Figure 11.1 shows a three-dimensional graphical representation of the resulting "traffic stream model," for a particular hypothetical desired

[3] In some of their work, Prigogine and Herman (1971, Section 4.4, esp. Fig. 4.8 and the related discussion) did permit positive lower bounds for the set of desired speeds, but for reasons that seem unclear at this point their attendant analysis did not identify the full two-parameter range of equilibrium solutions at higher densities.

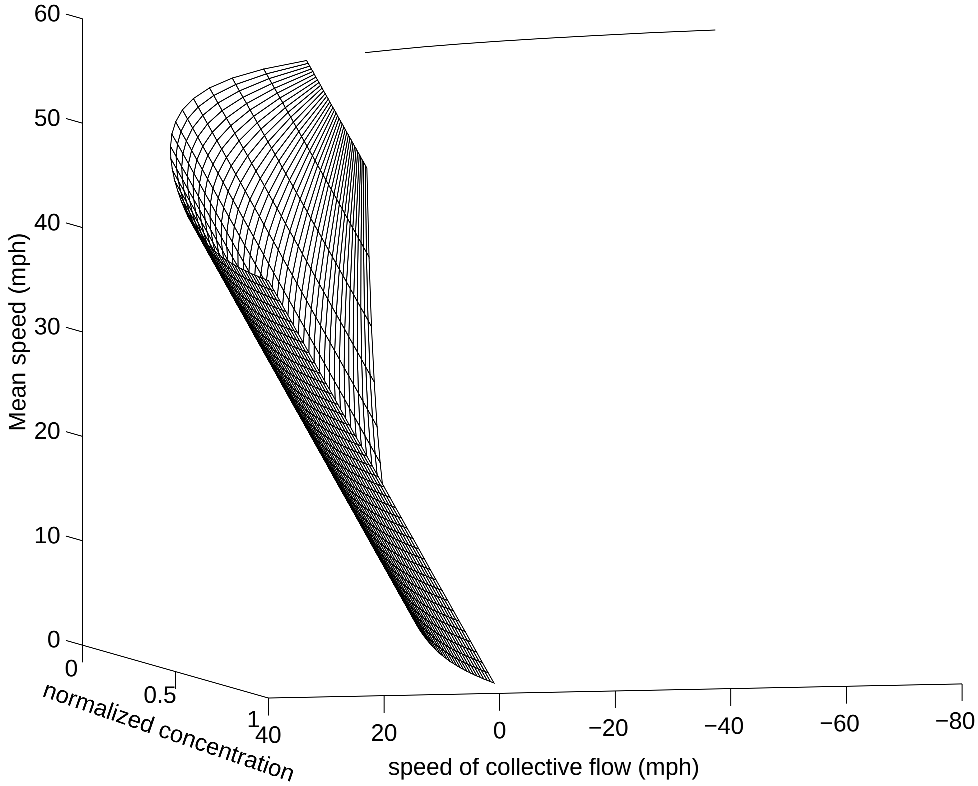

Figure 11-1 Dependence of the mean speed upon density normalized to jam density, $\eta = c/c_P$, for jam density $c_P = 200$ *vpm*, $P = 1-\eta$, $T = \tau\eta/(1-\eta)$, with $\tau = 0.003$ hours, and a uniform desired speed distribution from 40 to 80 mph.

speed distribution. See Nelson and Sopasakis (1998) for more details.

The significance of this three-dimensional presentation of a traffic stream model lies in the fact that it is consistent with the well-known tendency (e.g., Drake, Schofer and May, 1967) for traffic flow data to be widely scattered at high densities. The effort to explain this tendency has spawned a number of theories (e.g., Ceder, 1976; Hall, 1987; Disbro and Frame, 1989). The explanation in terms of an embedded collective flow seems possibly preferable to these, in that it derives from the kinetic theory, which is a well-known branch of traffic flow theory, as opposed to requiring some novel *ad hoc* theory.

Thus, the Prigogine-Herman kinetic equation has equilibrium solutions that both reproduce the observed bi-modal distribution of speeds at high densities, and provide traffic stream models that reproduce qualitatively the well-known result that at sufficiently high densities mean speeds and flows do not depend exclusively upon vehicular density. One certainly can envision more ambitious objectives for a kinetic theory of vehicular traffic than these two. Some possible such objectives are discussed further in Sections 4 and 5 below. However, given that the seminal Prigogine-Herman kinetic equation of vehicular traffic does accomplish at least these objectives, it seems appropriate to suggest them as minimal benchmarks that should be met by any alternative kinetic equations that might be proposed. In the following section some of the alternative kinetic equations that have been proposed, as described in the preceding subsection, are assessed against these benchmarks.

11.3 Other Kinetic Models

Both benchmarks suggested in the preceding subsection have to do with the equilibrium solutions of the kinetic equation of interest. The equilibrium solutions of the Paveri-Fontana (1975) generalization of the Prigogine-Herman kinetic equation, as described in Subsection 2.2, do not seem to have been definitively ascertained. Indeed, Helbing (1996), who has extensively applied the Paveri-Fontana kinetic model in his recent works on the kinetic theory of vehicular traffic, states, in regard to these equilibrium solutions, that “unfortunately it seems impossible to find an analytical expression ….” He then indicates that “empirical data and microsimulations” suggest these equilibrium solutions are “approximately a Gaussian.” Note that Gaussians are *not* bimodal. Thus, the Paveri-Fontana model does not seem to have been shown to satisfy either of the benchmarks suggested in the preceding subsection.

Phillips (1977, 1979) develops yet another kinetic equation that is an alternative to the original Prigogine-Herman kinetic model. However, this development seems predicated on a form of the corresponding equilibrium solution that ignores the considerations that led Prigogine and Herman to the “lower mode” of their bimodal equilibrium solution; cf. Eq. (4) of Phillips, 1979. Phillips compared (sketchily in Phillips, 1979, but exhaustively in Phillips, 1977) the equilibrium solution of his kinetic model against measured speed distributions. With one possibly important exception, the agreement seems reasonable. One therefore expects good agreement between the traffic stream model obtained theoretically from the equilibrium solution and that obtained observationally, although Phillips does not explicitly effect such comparisons. The exception is that a large amount of the data indicates a bimodal equilibrium solution; cf. Figs. 3 and 4 of Phillips, 1979, and numerous figures in Phillips, 1977. Thus, although the bimodal nature of an equilibrium solution is missed by the theoretical analysis, it is supported by the associated observations. In summary, it seems likely that the kinetic equation of Phillips (1979, 1977) meets the first benchmark suggested in the preceding section, and possible that a mathematical reassessment of its equilibrium solutions will reveal that it meets the second of these benchmarks. However, neither of these conclusions has yet been conclusively established.

Nelson (1995a) obtained a specific kinetic equation for purposes of providing a concrete illustration of his proposed general methodology for obtaining speeding-up (and slowing-down) terms based on first principles (i.e., appropriate mechanical and correlation models). In subsequent work (Nelson, Bui and Sopasakis, 1997) it was shown that this kinetic equation provides a theoretical traffic stream model that agrees well with classical traffic stream models, except near jam density. It has further been shown (Bui, Nelson and Sopasakis, 1996) that a simple modification of the underlying correlation model removes the incorrect behavior near jam density. Thus, this kinetic equation has been shown to meet the first of the benchmarks suggested in the preceding section. However, the equilibrium solutions of the kinetic equation of Nelson (1995a) are such that it clearly does *not* meet the second benchmark (i.e., does not predict scattered flow data under congestion). It is possible that the underlying mechanical model could be modified to attain this objective, but that has not been demonstrated.

Klar and Wegener (1999) use numerical techniques to obtain equilibrium solutions of their kinetic equations. They do not explicitly present corresponding traffic stream models. Their numerical equilibrium solutions do not display two modes. It might be difficult to obtain the lower mode, which typically appears as a delta function, by a strictly numerical approach.

Table I summarizes the status of the various kinetic models mentioned here, as regards their ability to meet the two benchmarks delineated in Subsection 2.3.

Table 11-I Status of various kinetic models with respect to the benchmarks of Subsection 11.2.3

Benchmark → Kinetic Model ↓	Bimodal equilibrium solutions?	Equilibrium with scattered flows at high densities?
Prigogine-Herman (1971)	yes	yes
Paveri-Fontana (1975)	?	?
Phillips (1977, 1979)	no	no
Nelson (1995a)	yes	no
Klar-Wegener (1999)	no?	no

11.4 Continuum Models from Kinetic Equations

Continuum models historically have played an important role in traffic flow theory. They have been obtained either by simply writing them as analogs of some corresponding fluid dynamical system (e.g., Kerner and Konhäuser, 1993), or by rational developments from some presumably more basic microscopic (e.g., car-following) model of traffic flow. In the latter case the continuum equations can be developed either directly from the underlying microscopic model that serves as the starting point, or a kinetic model can play an intermediary role between the microscopic and continuum models. For early examples of the former approach, through the steady-state solutions of car-following models, see numerous references cited in Nelson, 1995b. Nagel (1998) presents a more modern approach, through appropriate formal (“fluid-dynamical”) limits of particle-hopping models.

Here the primary interest is, of course, in approaches to continuum models that use a kinetic intermediary to the underlying microscopic model. Such approaches often (e.g., Helbing, 1995) follow the route of first taking the first few (one or two) low-order polynomial moments of the kinetic equation, then achieving closure via *ad hoc* approximations. An alternate approach, via certain formal asymptotic expansions (e.g., Hilbert or Chapman-Enskog expansions) is often used in the kinetic theory of gases (e.g., Grad, 1958). In this approach, the number of polynomial moments of the kinetic equation that are taken tend to be determined by the number of invariants that are defined by the dynamics of the microscopic model of the interaction between the constituent "particles" (vehicles, for traffic flow) of the system. This approach leads to a hierarchy of continuum models (e.g., the Euler/Navier-Stokes/Burnett/super-Burnett equations of fluid dynamics), as opposed to the single continuum equation that tends to result from formal limits of microscopic models. At all levels of this hierarchy the parameters of the resulting continuum model are expressed in terms of those of the underlying microscopic model.

Nelson and Sopasakis (1999) applied the Chapman-Enskog expansion to the Prigogine-Herman (1971) kinetic equation. In the region below the critical density described in Subsection 11.2 the lowest (zero) order expansion was found to be a Lighthill-Whitham (1955) continuum model, with associated traffic stream model corresponding to the one-parameter family of equilibrium solutions. The next highest (first-order) solution was found to be a diffusively corrected Lighthill-Whitham model,

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}\left[Q(c)\right] = \frac{\partial}{\partial x}\left[D(c)\frac{\partial c}{\partial x}\right], \qquad (11.2)$$

where now both the flow function $Q(c)$ and the diffusion coefficient $D(c)$ are known in terms of the density c and the parameters of the Prigogine-Herman kinetic model. This result is perhaps somewhat surprising, as one might reasonably have expected rather a continuum higher-order model of the type suggested by Payne (1971). Figure 11.2 illustrates how an initial discontinuity between an upstream higher-density region and a downstream lower-density region will tend to dissipate according to the diffusively corrected Lighthill-Whitham model, as opposed to the shock wave predicted by ordinary Lighthill-Whitham theory, which is given by Eq. (11.2) with $D \equiv 0$. See Nelson (2000) for more details of the example underlying this figure. Sopasakis (2000) has also developed the zero-order (again Lighthill-Whitham) and first-order Hilbert expansions, and the second-order Chapman-Enskog expansion, for the Prigogine-Herman kinetic model.

Interest in continuum models of traffic flow seems likely to continue, as applications exist within the space of resolution/fidelity/scale requirements for which continuum models are deemed most suitable. Along with this, interest in the use of kinetic models of vehicular traffic as a basis for continuum models seems likely to continue. For example, the venerable Lighthill-Whitham (1955) model is widely viewed as the most basic continuum model of traffic flow. But a suitable traffic stream model is an essential ingredient of the Lighthill-Whitham model. Thus, traffic stream models are an important part of continuum models, as well as being of interest in their own right. Therefore, both of the benchmarks demarcated in the preceding section can be viewed as related to the issue of how well a particular kinetic model performs in terms of providing a particularly low-order continuum model, specifically the Lighthill-Whitham model.

11.5 Direct Solution of Kinetic Equations

Along with the traditional application of kinetic models of vehicular traffic to rational development of continuum models from microscopic models, as described in the preceding section, modern computers permit consideration of the utility of kinetic models in their own right, rather than merely as tools that can be used to construct continuum models. In this respect, there are two substantive issues:

i) How can one solve kinetic equations, to obtain the distribution function (*f*)?

ii) Given this distribution function, what applications of it can usefully be made?

As regards the first issue, Hoogendoorn and Bovy (to appear) have employed Monte Carlo (i.e., simulation-based) techniques for the computational solution of a kinetic equation of vehicular traffic that builds upon the earlier work of Paveri-Fontana (1975). By contrast, in the kinetic theory of gases there exists a significant body of knowledge (e.g., Neunzert and Struckmeier, 1995, and other works cited therein) relative to the deterministic computational solution of kinetic equations. This knowledge base undoubtedly could be invaluable in attempting to develop similar capabilities for vehicular traffic, but the equations are sufficiently different from those arising in the kinetic theory of gases so that considerable further development is likely to be necessary. This

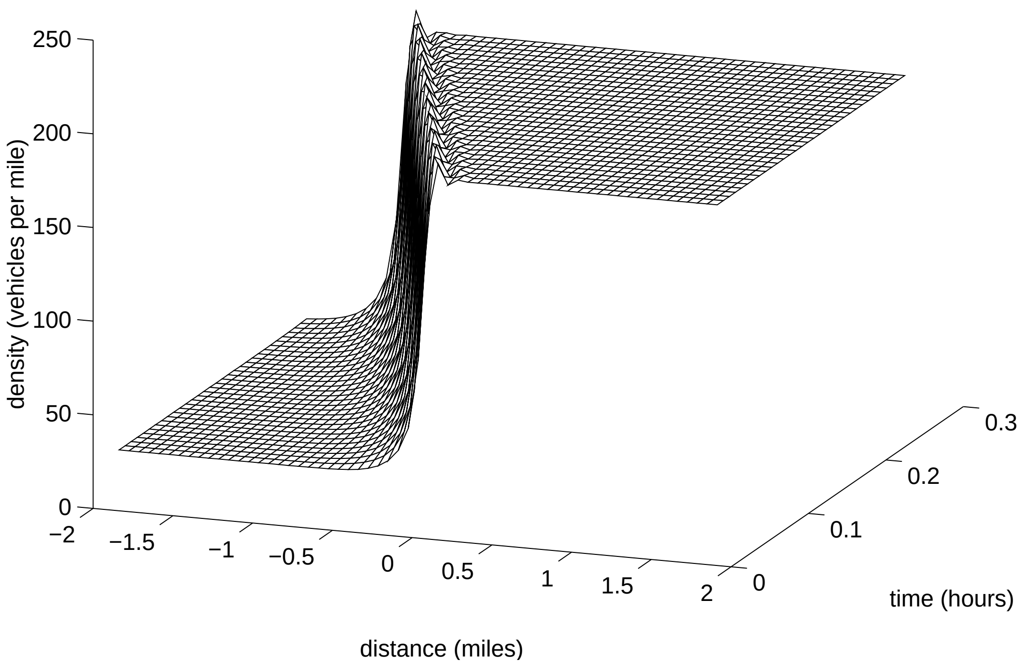

Figure 11-2 Evolution of the flow, according to a diffusively corrected Lighthill-Whitham model, from initial conditions consisting of 190 vpm downstream of *x=0* and 30 vpm upstream. See Nelson (2000) for details of the traffic stream model (flow function) and diffusion coefficient.

development is unlikely to occur in the absence of a relatively clear vision as to the uses that would be made of it. Therefore, the focus here primarily will be on the second of these issues.

Any consideration of applications of the distribution function of a kinetic theory requires a consideration of its interpretation. It is a statistical distribution function. The traditional interpretation of such a distribution function is that it describes the frequency with which certain properties occur among samples drawn from some sample space. In the kinetic theory of vehicular traffic, the samples are vehicles, and the properties of interest are the positions and speeds of the vehicles; however, there are two fundamentally different possible interpretations of the underlying sample space:

1. **Single-instance sampling:** The sample space consists of all vehicles present on a specified road network at a specific designated time.
2. **Ensemble sampling:** The sample space consists of all vehicles present, at a designated time, on one of an *ensemble* of identical road networks.

For example, the Houston freeway network at 5:00 p.m. on Wednesday, July 15, 1998 would be a reasonable sample space for single-instance sampling. On the other hand, the Houston freeway network at 5:00 p.m. on all midweek workdays during 1998 for which dry weather conditions prevailed would be a reasonable approximation of a sample space suitable for ensemble sampling.

The difference between these two interpretations is subtle, but it has profound consequences. Traffic theorists normally tend to think in terms that are most consistent with single-instance sampling. But any attempt to apply the kinetic theory within that interpretation implies the intention to predict, at some level of approximation, the evolution of traffic for that specific instance, given suitable initial and boundary conditions for the distribution function. It seems somewhat questionable that this is attainable over any significant duration. (The ``rolling horizon'' approach often applied to prediction of traffic flow is a tacit admission of the significance of this issue.) On the other hand, the ensemble sampling interpretation implies only the intent to predict the likelihood with which various outcomes will occur. This intuitively seems much more achievable (cf. p. 10 of Asimov, 1988). Thus the more subtle ensemble sampling interpretation leads to an apparently more achievable objective than does the more obvious single-instance interpretation. For that reason, the ensemble sampling interpretation seems more likely to lead to potential direct applications of the kinetic theory.

The fundamental advantage of kinetic models over continuum models is that the kinetic distribution function provides an estimate of the variability (over various instances of the ensemble, under the ensemble-sampling interpretation) of densities and speed at specific times and locations, whereas continuum models provide estimates of only the mean (presumably over the ensemble) of these quantities. If the quantity (function of position and time) of interest in a particular application is not highly variable between instances within the ensemble, or if that variability is not of interest, then presumably one should choose a continuum model, or perhaps an even more highly aggregated model. On the other hand, if this variability is both of significant magnitude and important to the issue under study, then kinetic models might be a useful alternative to the computationally more expensive possibility of running a sufficiently large number of microscopic simulations so as to capture the nature of the variation between instances.

Some specific instances of quantities for which variability might be of significant interest are travel times and driving cycles. The latter require knowledge about the statistical distribution of accelerations, as well as velocities and densities, but such acceleration information is inherent in the distribution function, along with the mechanical and correlation models that underlie any kinetic model. In fact, kinetic models seem to be the natural connecting link between continuum models, which provide the "cross sectional" view of traffic that most transportation planning is based on, and the "longitudinal" view that underlies the standard driving cycle approach to estimation of fuel emissions (cf. Carson and Austin, 1997).

The crucial question underlying any potential application of kinetic models is whether a kinetic model can be found that has sufficient fidelity and resolution for the particular application, and that can be solved on the necessary scale using available computational resources. The answer to that clearly depends upon the specific details of the particular application, and any such proposed application of a specific kinetic model must be validated against actual observations. However, data of sufficiently high quality to permit such validations are both rare and expensive to obtain. Under these circumstances, it seems appropriate to use microscopic models (e.g., cellular automata) as a framework within which initially to vet proposed kinetic models.

Specifically, it seems worthwhile to employ microscopic models to study the following:

HYPOTHESIS: *The multiparameter family of equilibrium solutions of the Prigogine-Herman kinetic model found by Nelson and Sopasakis (1998), with its attendant traffic stream surface (rather than the classical curve), reflects the fact that actual traffic has a number of spatially homogeneous equilibrium states (with different average speeds) corresponding to the same density.*

If this hypothesis is true, then presumably different initial configurations of a traffic stream have the possibility to approach different states in the long-time limit, even though their densities are the same on the macroscopic scale. Results reported by Nagel (1996, esp. Sec. V) tend to confirm this hypothesis.

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