MACROSCOPIC FLOW MODELS

BY JAMES C. WILLIAMS[9]

[9] Associate Professor, Department of Civil Engineering, University of Texas at Arlington, Box 19308, Arlington, TX 76019-0308.

CHAPTER 6 - Frequently used Symbols

Note to reader: The symbols used in Chapter 6 are the same as those used in the original sources. Therefore, the reader is cautioned that the same symbol may be used for different quantities in different sections of this chapter. The symbol definitions below include the sections in which the symbols are used if the particular symbol definition changes within the chapter or is a definition particular to this chapter. In each case, the symbols are defined as they are introduced within the text of the chapter. Symbol units are given only where they help define the quantity; in most cases, the units may be in either English or metric units as necessary to be consistent with other units in a relation.

A	=	area of town (Section 6.2.1)
c	=	capacity (vehicles per unit time per unit width of road) (Section 6.2.1)
D	=	delay per intersection (Section 6.2.2)
f	=	fraction of area devoted to major roads (Section 6.1.1)
f	=	fraction of area devoted to roads (Section 6.2.1)
f	=	number of signalized intersections per mile (Section 6.2.2)
f_r	=	fraction of moving vehicles in a designated network (Section 6.3)
f_s	=	fraction of stopped vehicles in a designated network (Section 6.3)
$f_{s,min}$	=	minimum fraction of vehicles stopped in a network (Section 6.4)
I	=	total distance traveled per unit area, or traffic intensity (pcu/hour/km) (Sections 6.1.1 and 6.2.3)
J	=	fraction of roadways used for traffic movement (Section 6.2.1)
K	=	average network concentration (ratio of the number of vehicles in a network and the network length, Section 6.4)
K_j	=	jam network concentration (Section 6.4)
N	=	number of vehicles per unit time that can enter the CBD (Section 6.2.1)
n	=	quality of traffic indicator (two-fluid model parameter, Section 6.3)
Q	=	capacity (pcu/hr) (Section 6.2.2)
Q	=	average network flow, weighted average over all links in a designated network (Section 6.4)
q	=	average flow (pcu/hr)
R	=	road density, i.e., length or area of roads per unit area (Section 6.2.3)
r	=	distance from CBD
T	=	average travel time per unit distance, averaged over all vehicles in a designated network (Section 6.3)
T_m	=	average minimum trip time per unit distance (two-fluid model parameter, Section 6.3)
T_r	=	average moving (running) time per unit distance, averaged over all vehicles in a designated network (Section 6.3)
T_s	=	average stopped time per unit distance, averaged over all vehicles in a designated network (Section 6.3)
V	=	average network speed, averaged over all vehicles in a designated network (Section 6.4)
V_f	=	network free flow speed (Section 6.4)
V_m	=	average maximum running speed (Section 6.2.3)
V_r	=	average speed of moving (running) vehicles, averaged over all in a designated network (Section 6.3)
v	=	average speed
v	=	weighted space mean speed (Section 6.2.3)
v_r	=	average running speed, i.e., average speed while moving (Section 6.2.2)
w	=	average street width
α	=	Zahavi's network parameter (Section 6.2.3)
λ	=	g/c time, i.e. ration of effective green to cycle length

6.
MACROSCOPIC FLOW MODELS

Mobility within an urban area is a major component of that area's quality of life and an important issue facing many cities as they grow and their transportation facilities become congested. There is no shortage of techniques to improve traffic flow, ranging from traffic signal timing optimization (with elaborate, computer-based routines as well as simpler, manual, heuristic methods) to minor physical changes, such as adding a lane by the elimination of parking. However, the difficulty lies in evaluating the effectiveness of these techniques. A number of methods currently in use, reflecting progress in traffic flow theory and practice of the last thirty years, can effectively evaluate changes in the performance of an intersection or an arterial. But a dilemma is created when these individual components, connected to form the traffic network, are dealt with collectively.

The need, then, is for a consistent, reliable means to evaluate traffic performance in a network under various traffic and geometric configurations. The development of such performance models extends traffic flow theory into the network level and provides traffic engineers with a means to evaluate system-wide control strategies in urban areas. In addition, the quality of service provided to the motorists could be monitored to evaluate a city's ability to manage growth. For instance, network performance models could also be used by a state agency to compare traffic conditions between cities in order to more equitably allocate funds for transportation system improvements.

The performance of a traffic system is the response of that system to given travel demand levels. The traffic system consists of the network topology (street width and configuration) and the traffic control system (e.g., traffic signals, designation of one- and two-way streets, and lane configuration). The number of trips between origin and destination points, along with the desired arrival and/or departure times comprise the travel demand levels. The system response, i.e., the resulting flow pattern, can be measured in terms of the level of service provided to the motorists. Traffic flow theory at the intersection and arterial level provides this measurement in terms of the three basic variables of traffic flow: speed, flow (or volume), and concentration. These three variables, appropriately defined, can also be used to describe traffic at the network level. This description must be one that can overcome the intractabilities of existing flow theories when network component interactions are taken into account.

The work in this chapter views traffic in a network from a macroscopic point of view. Microscopic analyses run into two major difficulties when applied to a street network:

1) Each street block (link) and intersection are modeled individually. A proper accounting of the interactions between adjacent network components (particularly in the case of closely spaced traffic signals) quickly leads to intractable problems.

2) Since the analysis is performed for each network component, it is difficult to summarize the results in a meaningful fashion so that the overall network performance can be evaluated.

Simulation can be used to resolve the first difficulty, but the second remains; traffic simulation is discussed in Chapter 10.

The Highway Capacity Manual (Transportation Research Board 1994) is the basic reference used to evaluate the quality of traffic service, yet does not address the problem at the network level. While some material is devoted to assessing the level of service on arterials, it is largely a summation of effects at individual intersections. Several travel time models, beginning with the travel time contour map, are briefly reviewed in the next section, followed by a description of general network models in Section 6.2. The two-fluid model of town traffic, also a general network model, is discussed separately in Section 6.3 due to the extent of the model's development through analytical, field, and simulation studies. Extensions of the two-fluid model into general network models are examined in Section 6.4, and the chapter references are in the final section.

6.1 Travel Time Models

Travel time contour maps provide an overview of how well a street network is operating at a specific time. Vehicles can be dispatched away from a specified location in the network, and each vehicle's time and position noted at desired intervals.

Contours of equal travel time can be established, providing information on the average travel times and mean speeds over the network. However, the information is limited in that the travel times are related to a single point, and the study would likely have to be repeated for other locations. Also, substantial resources are required to establish statistical significance. Most importantly, though, is that it is difficult to capture network performance with only one variable (travel time or speed in this case), as the network can be offering quite different levels of service at the same speed.

This type of model has be generalized by several authors to estimate average network travel times (per unit distance) or speeds as a function of the distance from the central business district (CBD) of a city, unlike travel time contour maps which consider only travel times away from a specific point.

6.1.1 General Traffic Characteristics as a Function of the Distance from the CBD

Vaughan, Ioannou, and Phylactou (1972) hypothesized several general models using data from four cities in England. In each case, general model forms providing the best fit to the data were selected. Traffic intensity (I, defined as the total distance traveled per unit area, with units of pcu/hour/km) tends to decrease with increasing distance from the CBD,

$$I = A\exp\left(-\sqrt{r/a}\right) \tag{6.1}$$

where r is the distance from the CBD, and A and a are parameters. Each of the four cities had unique values of A and a, while A was also found to vary between peak and off-peak periods. The data from the four cities is shown in Figure 6.1. A similar relation was found between the fraction of the area which is major road (f) and the distance from the CBD,

$$f = B\exp\left(-\sqrt{r/b}\right)\ , \tag{6.2}$$

where b and B are parameters for each town. Traffic intensity and fraction of area which is major road were found to be linearly related, as was average speed and distance from the CBD. Since only traffic on major streets is considered, these

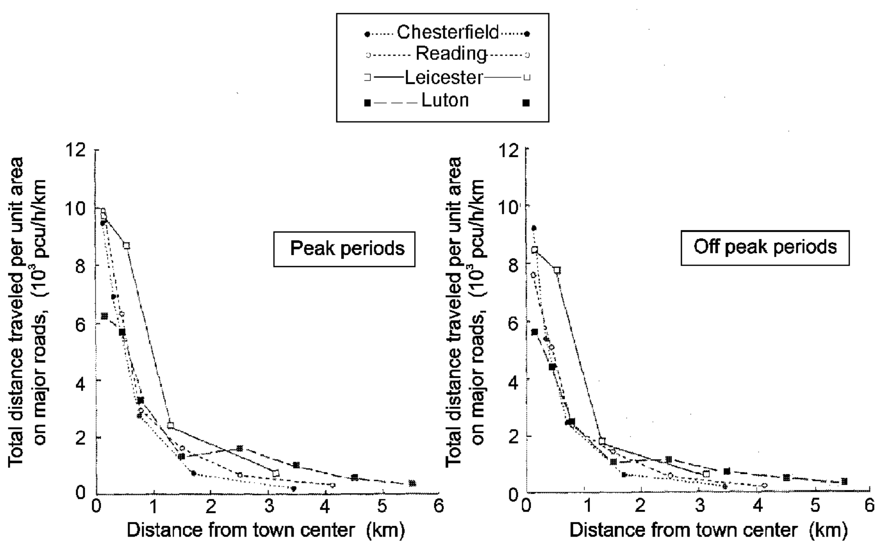


Figure 6.1
Total Vehicle Distance Traveled Per Unit Area on Major Roads as a Function of the Distance from the Town Center (Vaughan et al. 1972).

results are somewhat arbitrary, depending on the streets selected as major.

6.1.2 Average Speed as a Function of Distance from the CBD

Branston (1974) investigated five functions relating average speed (v) to the distance from the CBD (r) using data collected by the Road Research Laboratory (RRL) in 1963 for six cities in England. The data was fitted to each function using least-squares regression for each city separately and for the aggregated data from all six cities combined. City centers were defined as the point where the radial streets intersected, and the journey speed in the CBD was that found within 0.3 km of the selected center. Average speed for each route section was found by dividing the section length by the actual travel time (miles/minute). The five selected functions are described below, where a, b, and c are constants estimated for the data. A power curve,

$$v = a r^{b} \tag{6.3}$$

was drawn from Wardrop's work (1969), but predicts a zero speed in the city center (at $r = 0$). Accordingly, Branston also fitted a more general form,

$$v = c + ar^{b} \quad , \tag{6.4}$$

where c represents the speed at the city center.

Earlier work by Beimborn (1970) suggested a strictly linear form, up to some maximum speed at the city edge, which was defined as the point where the average speed reached its maximum (i.e., stopped increasing with increasing distance from the center). None of the cities in Branston's data set had a clear maximum limit to average speed, so a strict linear function alone was tested:

$$v = a + br \quad . \tag{6.5}$$

A negative exponential function,

$$v = a - be^{-cr} \quad , \tag{6.6}$$

had been fitted to data from a single city (Angel and Hyman 1970). The negative exponential asymptotically approaches some maximum average speed.

The fifth function, suggested by Lyman and Everall (1971),

$$v = \frac{1 + b^2 r^2}{a + cb^2 r^2} \tag{6.7}$$

also suggested a finite maximum average speed at the city outskirts. It had originally be applied to data for radial and ring roads separately, but was used for all roads here.

Two of the functions were quickly discarded: The linear model (Equation 6.5) overestimated the average speed in the CBDs by 3 to 4 km/h, reflecting an inability to predict the rapid rise in average speed with increasing distance from the city center. The modified power curve (Equation 6.4) estimated negative speeds in the city centers for two of the cities, and a zero speed for the aggregated data. While obtaining the second smallest sum of squares (negative exponential, Equation 6.6, had the smallest), the original aim of using this model (to avoid the estimation of a zero journey speed in the city center) was not achieved.

The fitted curves for the remaining three functions (negative exponential, Equation 6.6; power curve, Equation 6.3; and Lyman and Everall, Equation 6.7) are shown for the data from Nottingham in Figure 6.2. All three functions realistically predict a leveling off of average speed at the city outskirts, but only the Lyman-Everall function indicates a leveling off in the CBD. However, the power curve showed an overall better fit than the Lyman-Everall model, and was preferred.

While the negative exponential function showed a somewhat better fit than the power curve, it was also rejected because of its greater complexity in estimation (a feature shared with the Lyman-Everall function). Truncating the power function at measured downtown speeds was suggested to overcome its drawback of estimating zero speeds in the city center. The complete data set for Nottingham is shown in Figure 6.3, showing the fitted power function and the truncation at $r = 0.3$ km.

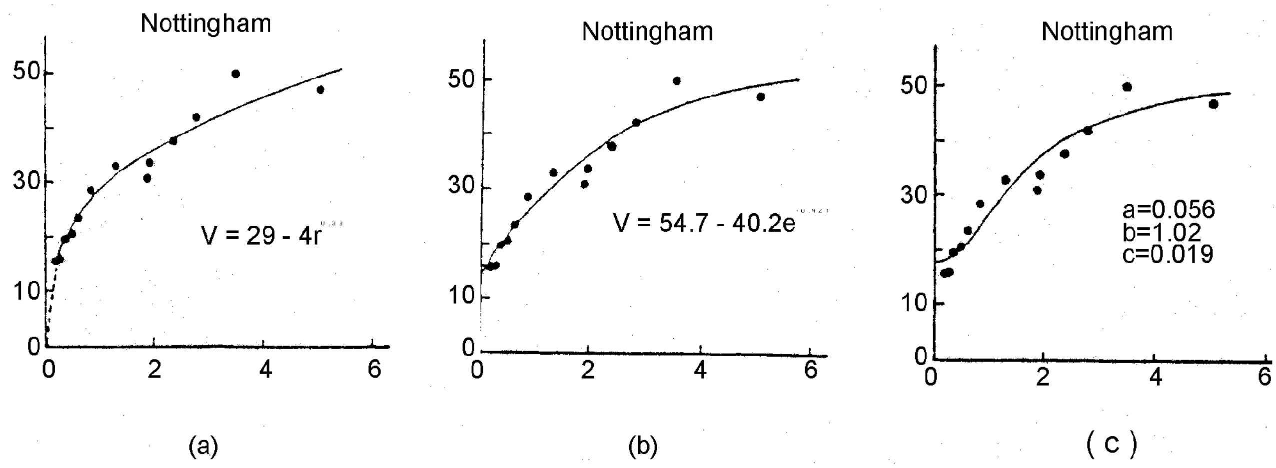


Figure 6.2
Grouped Data for Nottingham Showing Fitted a) Power Curve, b) Negative Exponential Curve, and c) Lyman-Everall Curve (Branston 1974, Portions of Figures 1A, 1B, and 1C).

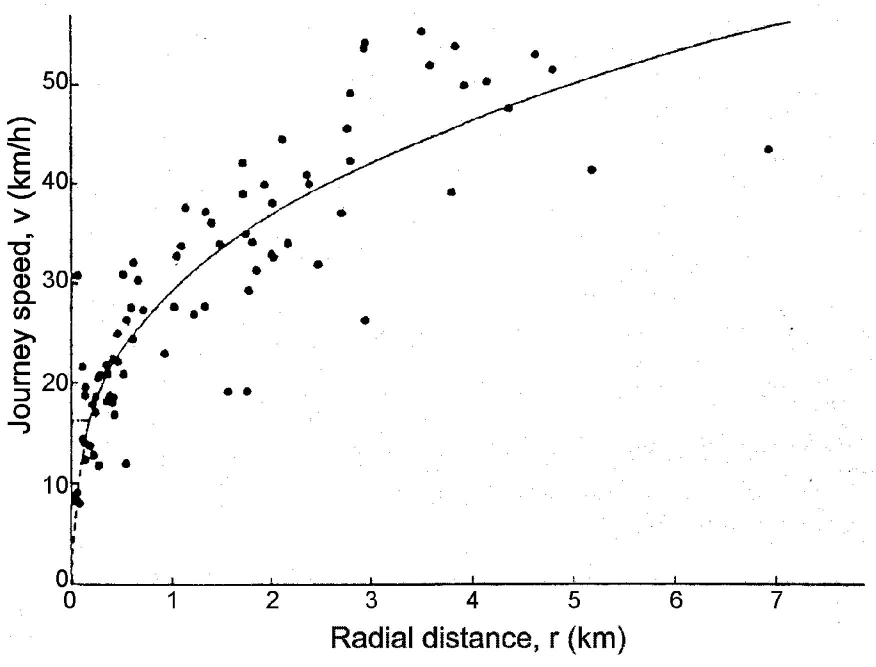


Figure 6.3
Complete Data Plot for Nottingham; Power Curve Fitted to the Grouped Data (Branston 1974, Figure 3).

If the data is broken down by individual radial routes, as shown in Figure 6.4, the relation between speed and distance from the city center is stronger than when the aggregated data is examined.

Hutchinson (1974) used RRL data collected in 1967 from eight cities in England to reexamine Equations 6.3 and 6.6 (power curve and negative exponential) with an eye towards simplifying them.

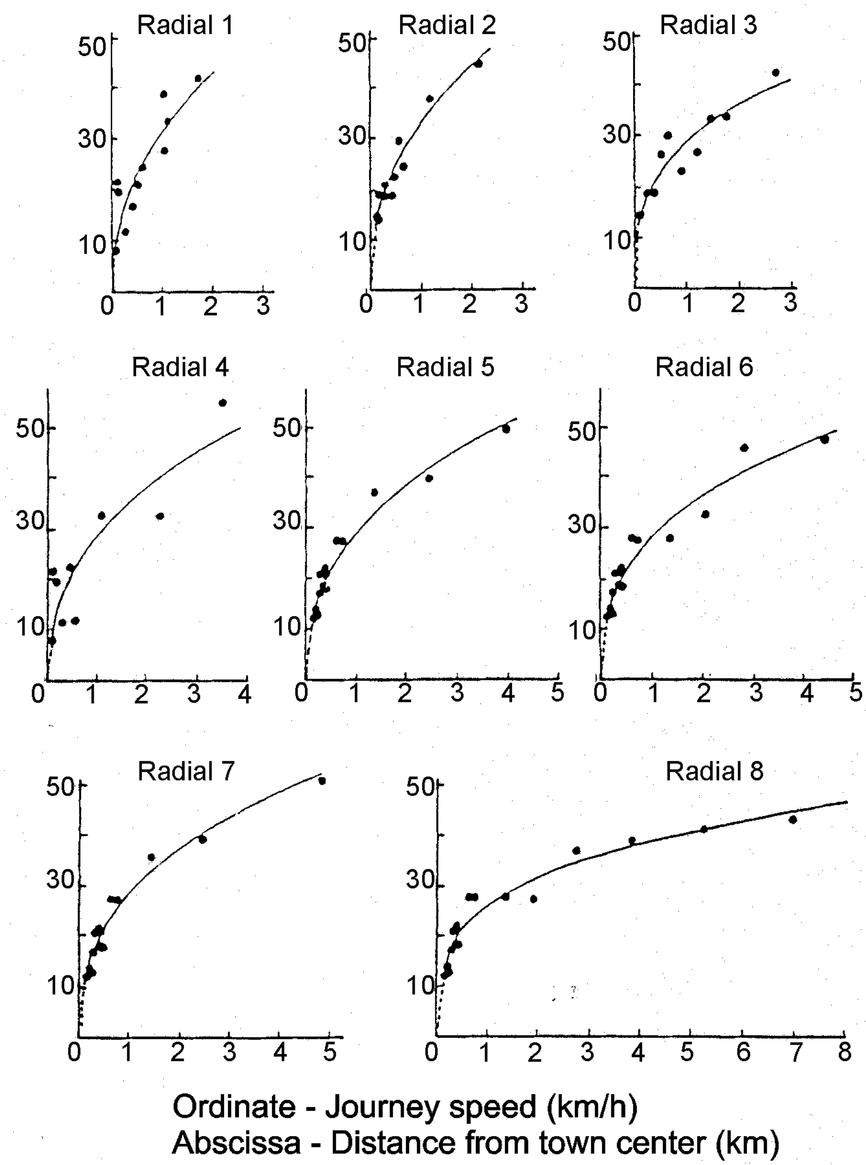


Figure 6.4
Data from Individual Radial Routes in Nottingham,
Best Fit Curve for Each Route is Shown (Branston 1974, Figure 4).

The exponents of the power functions fitted by Branston (1974) fell in the range 0.27 to 0.36, suggesting the following simplification

$$v = k r^{1/3} \tag{6.8}$$

When fitted to Branston's data, there was an average of 18 percent increase in the sum of squares. The other parameter, *k*, was found to be significantly correlated with the city population, with different values for peak and off-peak conditions. The parameter *k* was found to increase with increasing population, and was 9 percent smaller in the peak than in the off-peak.

In considering the negative exponential model (Equation 6.6), Hutchinson reasoned that average speed becomes less characteristic of a city with increasing *r*, and, as such, it would be reasonable to select a single maximum limit for *v* for every city. Assuming that any speed between 50 and 75 km/h would make little difference, Hutchinson selected 60 km/h, and

$$v = 60 - a e^{-r/R} \quad . \tag{6.9}$$

Hutchinson found that this model raised the sum of squares by 30 percent (on the average) over the general form used by Branston. *R* was found to be strongly correlated with the city population, as well as showing different averages with peak and off-peak conditions, while *a* was correlated with neither the city population nor the peak vs. off-peak conditions. The difference in the *R*s between peak and off-peak conditions (30 percent higher during peaks) implies that low speeds spread out over more of the network during the peak, but that conditions in the city center are not significantly different. Hutchinson (1974) used RRL data collected in 1967 from eight cities in England to reexamine Equations 6.3 and 6.6 (power curve and negative exponential) with an eye towards simplifying them.

6.2 General Network Models

A number of models incorporating performance measures other than speed have been proposed. Early work by Wardrop and Smeed (Wardrop 1952; Smeed 1968) dealt largely with the development of macroscopic models for arterials, which were later extended to general network models.

6.2.1 Network Capacity

Smeed (1966) considered the number of vehicles which can "usefully" enter the central area of a city, and defined *N* as the number of vehicles per unit time that can enter the city center. In general, *N* depends on the general design of the road network, width of roads, type of intersection control, distribution of destinations, and vehicle mix. The principle variables for towns with similar networks, shapes, types of control, and vehicles are: *A*, the area of the town; *f*, the fraction of area devoted to roads; and *c*, the capacity, expressed in vehicles per unit time per unit width of road (assumed to be the same for all roads). These are related as follows:

$$N = \alpha f c \sqrt{A}, \tag{6.10}$$

where α is a constant. General relationships between *f* and $(N/c\sqrt{A})$ for three general network types (Smeed 1965) are shown in Figure 6.5. Smeed estimated a value of *c* (capacity per unit width of road) by using one of Wardrop's speed-flow equations for central London (Smeed and Wardrop 1964),

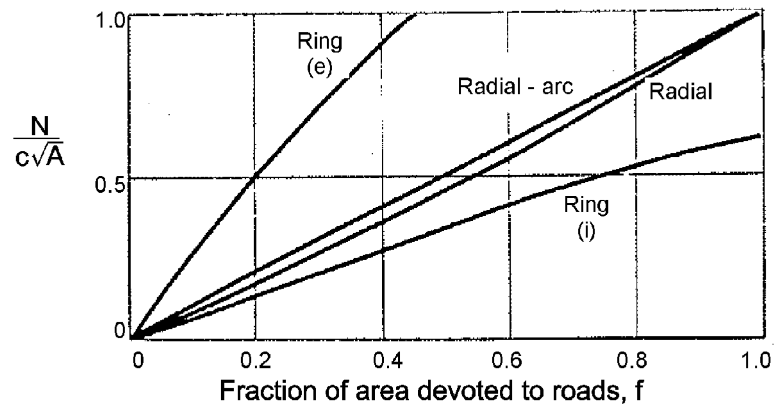
$$q = 2440 - 0.220\, v^3 \quad , \tag{6.11}$$

where *v* is the speed in kilometers/hour, and *q* the average flow in pcus/hour, and divided by the average road width, 12.6 meters,

$$c = 58.2 - 0.00524\, v^3 \ . \tag{6.12}$$

A different speed-flow relation which provided a better fit for speeds below 16 km/h resulted in $c = 68 - 0.13\, v^2$ (Smeed 1963).

Equation 6.12 is shown in Figure 6.6 for radial-arc, radial, and ring type networks for speeds of 16 and 32 km/h. Data from several cities, also plotted in Figure 6.6, suggests that αc=30,



Note: (e=excluding area of ring road, I=including area of ring road)

Figure 6.5
Theoretical Capacity of Urban Street Systems (Smeed 1966, Figure 2).

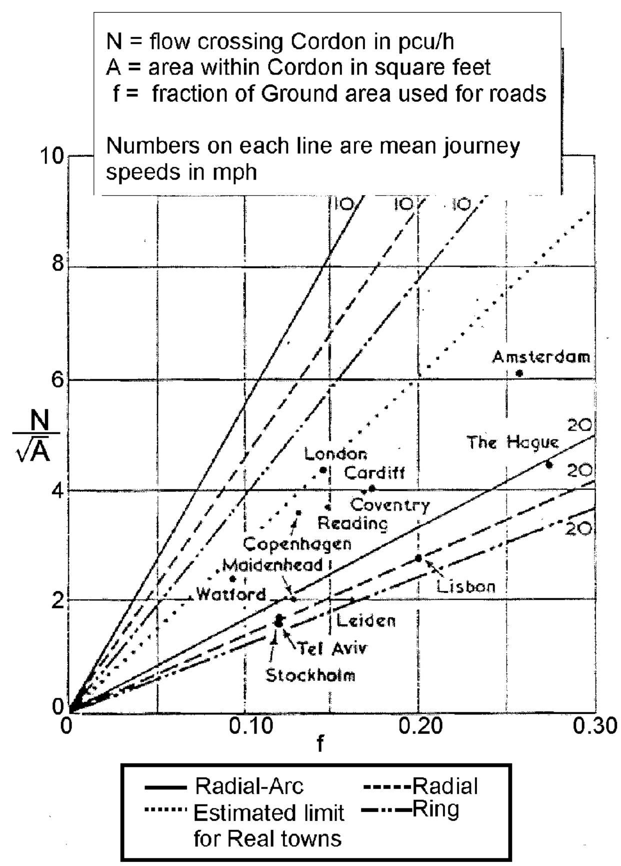


Figure 6.6
Vehicles Entering the CBDs of Towns Compared with the Corresponding Theoretical Capacities of the Road Systems (Smeed 1966, Figure 4).

and using the peak period speed of 16 km/h in central London, Equation 6.10 becomes

$$N = \left(33 - 0.003\, v^3\right)\; f\sqrt{A}\ , \qquad (6.13)$$

where v is in miles/hour and A in square feet. It should be noted that f represents the fraction of total area usefully devoted to roads. An alternate formulation (Smeed 1968) is

$$N = \left(33 - 0.003\, v^3\right)\; J f \sqrt{A} \qquad (6.14)$$

where f is the fraction of area actually devoted to roads, while J is the fraction of roadways used for traffic movement. J was found to range between 0.22 and 0.46 in several cities in England. The large fraction of unused roadway is mostly due to the uneven distribution of traffic on all streets. The number of vehicles which can circulate in a town depends strongly on their average speed, and is directly proportional to the area of usable roadway. For a given area devoted to roads, the larger the central city, the smaller the number of vehicles which can circulate in the network, suggesting that a widely dispersed town is not necessarily the most economical design.

6.2.2 Speed and Flow Relations

Thomson (1967b) used data from central London to develop a linear speed-flow model. The data had been collected once every two years over a 14-year period by the RRL and the Greater London Council. The data consisted of a network-wide average speed and flow each year it was collected. The average speed was found by vehicles circulating through central London on predetermined routes. Average flows were found by first converting measured link flows into equivalent passenger carunits, then averaging the link flows weighted by their respective link lengths. Two data points (each consisting of an average speed and flow) were found for each of the eight years the data was collected: peak and off-peak.

Plotting the two points for each year, Figure 6.7, resulted in a series of negatively sloped trends. Also, the speed-flow capacity (defined as the flow that can be moved at a given speed) gradually increased over the years, likely due to geometric and traffic control improvements and "more efficient vehicles." This indicated that the speed-flow curve had been gradually changing, indicating that each year's speed and flow fell on different curves. Two data points were inadequate to determine the shape of the curve, so all sixteen data points were used by accounting for the

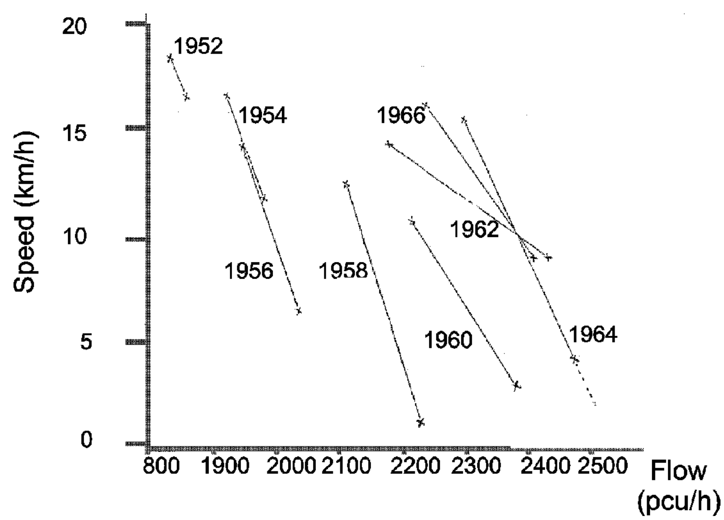


Figure 6.7
Speeds and Flows in Central London, 1952-1966, Peak and Off-Peak (Thomson 1967b, Figure 11)

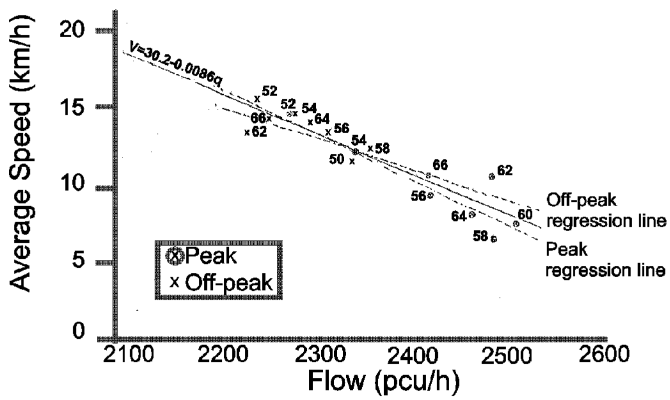
changing capacity of the network, and scaling each year's flow measurement to a selected base year. Using linear regression, the following equation was found:

$$v = 30.2 - 0.0086\,q \qquad (6.15)$$

where v is the average speed in kilometers/hour and q is the average flow in pcu/hour. This relation is plotted in Figure 6.8.

The equation implies a free-flow speed of about 48.3 km/h however, there were no flows less than 2200 pcu/hour in the historical data.

Thomson used data collected on several subsequent Sundays (Thomson 1967a) to get low flow data points. These are reflected in the trend shown in Figure 6.9.Also shown is a curve developed by Smeed and Wardrop using data from a single year only.



Note: Scaled to 1964 equivalent flows.

Figure 6.8
Speeds and Scaled Flows, 1952-1966 (Thomson 1967b, Figure 2).

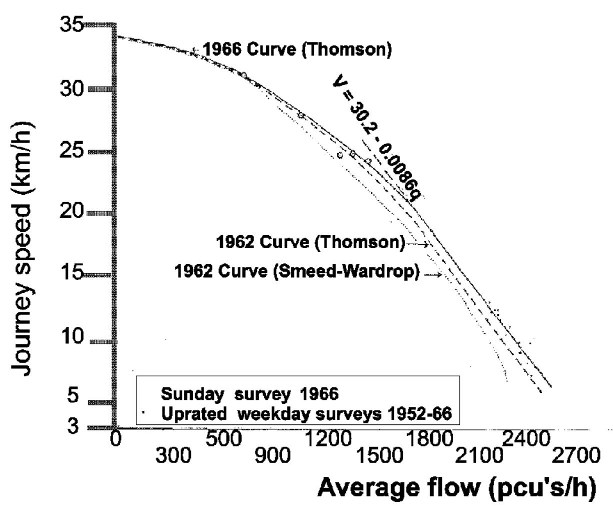


Figure 6.9
Estimated Speed-Flow Relations in Central London (Main Road Network) (Thomson 1967b, Figure 4).

The selected area of central London could be broken into inner and outer zones, distinguished principally by traffic signal densities, respectively 7.5 and 3.6 traffic signals per route-mile. Speed and flow conditions were found to be significantly different between the zones, as shown in Figure 6.10, and for the inner zone,

$$v = 24.3 - 0.0075\,q \quad , \tag{6.16}$$

and for the outer zone,

$$v = 34.0 - 0.0092\,q \quad . \tag{6.17}$$

Wardrop (1968) directly incorporated average street width and average signal spacing into a relation between average speed and flow, where the average speed includes the stopped time. In order to obtain average speeds, the delay at signalized intersections must be considered along with the running speed between the controlled intersections, where running speed is defined as the average speed while moving. Since speed is the inverse of travel time, this relation can be expressed as:

$$\frac{1}{v} = \frac{1}{v_r} + f\,d \quad , \tag{6.18}$$

where v is the average speed in mi/h, v_r, the running speed in mi/h, d the delay per intersection in hours, and f the number of signalized intersections per mile. Assuming $v_r = a(1\text{-}q/Q)$ and $d = b/(1\text{-}q/\lambda s)$, where q is the flow in pcu/hr, Q is the capacity in pcu/hr, λ is the g/c time, and s is the saturation flow in pcu/hr, and combining into Equation 6.18,

$$\frac{1}{v} = \frac{1}{a\,(1 - q/Q)} + \frac{f\,b}{1 - q/\lambda s} \tag{6.19}$$

Using an expression for running speed found for central London (Smeed and Wardrop 1964; RRL 1965),

$$v_r = 31 - \frac{0.70q + 430}{3w} \tag{6.20}$$

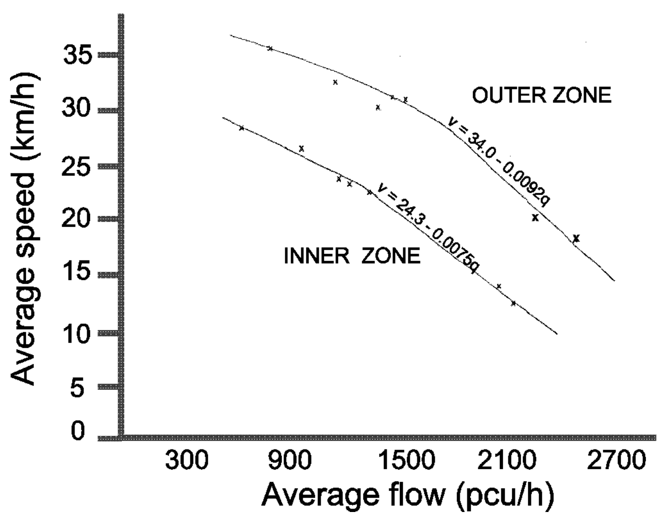


Figure 6.10
Speed-Flow Relations in Inner and Outer Zones of Central Area (Thomson 1967a, Figure 5).

where w is the average roadway width in feet, and an average street width of 42 feet (in central London), Equation 6.20 becomes $v_r = 28 - 0.0056\, q$, or 24 mi/h, whichever is less. The coefficient of q was modified to 0.0058 to better fit the observed running speed.

Using observed values of 0.038 hours/mile stopped time, 2180 pcu/hr flow, and 2610 pcu/hr capacity, the numerator of the second term of Equation 6.19 (fb) was found to be 0.0057. Substituting the observed values into Equation 6.19,

$$\frac{1}{v} = \frac{1}{28 - 0.0058\, q} + \frac{0.0057}{1 - \dfrac{q}{2610}} \quad .$$

Simplifying,

$$\frac{1}{v} = \frac{1}{28 - 0.0058\, q} + \frac{1}{197 - 0.0775\, q} \qquad (6.21)$$

Revising the capacity to 2770 pcu/hour (to reflect 1966 data), thus changing the coefficient of q in the second term of Equation 6.21 to 0.071, this equation provided a better fit than Thomson's linear relation (Thomson 1967b) and recognizes the known information on the ultimate capacity of the intersections.

Generalizing this equation for urban areas other than London, and knowing that the average street width in central London was 12.6 meters, the running speed can be written

$$\begin{aligned} v_r &= 31 - \frac{430}{3w} - \frac{a\,q}{w} \\ &= 31 - \frac{140}{w} - \frac{a\,q}{w} \quad . \end{aligned}$$

Since $a/w = 0.0058$ when $w = 42$ by Equation 6.21, a=0.0244, then

$$v_r = 31 - \frac{140}{w} - 0.0244\,\frac{q}{w} \quad . \qquad (6.22)$$

For the delay term, five controlled intersections per mile and a g/c of 0.45 were found for central London. Additionally, the intersection capacity was assumed to be proportional to the average stop line width, given that it is more than 5 meters wide (RRL 1965), which was assumed to be proportional to the roadway width. The general form for the delay equation (second term of Equation 6.21) is

$$f d = \frac{f b}{1 - q/k\lambda w} \qquad (6.23)$$

where k is a constant. For central London, $w = 42$, $\lambda = 0.45$, and $k\lambda w = Q = 2770$, thus $k = 147$, yielding

$$f d = \frac{f b}{1 - q/147\,\lambda w} \qquad (6.24)$$

Given that $f = 5$ signals/mile and $fb = 0.00507$ for central London, $b = 0.00101$, yielding

$$f d = \frac{f}{1000 - 6.8\, q/\lambda w} \quad . \qquad (6.25)$$

Combining, then, for the general equation for average speed:

$$\frac{1}{v} = \frac{1}{31 - \dfrac{140}{w} - 0.0244\dfrac{q}{w}} + \frac{f}{1000 - 6.8\dfrac{q}{\lambda w}} \quad . \qquad (6.26)$$

The sensitivity of Equation 6.26 to flow, average street width, number of signalized intersections per mile, and the fraction of green time are shown in Figures 6.11, 6.12, and 6.13. By calibrating this relation on geometric and traffic control features in the network, Wardrop extended the usefulness of earlier speed flow relations. While fitting nicely for central London, the applicability of this relation to other cities in its generalized format (Equation 6.26) is not shown, due to a lack of available data.

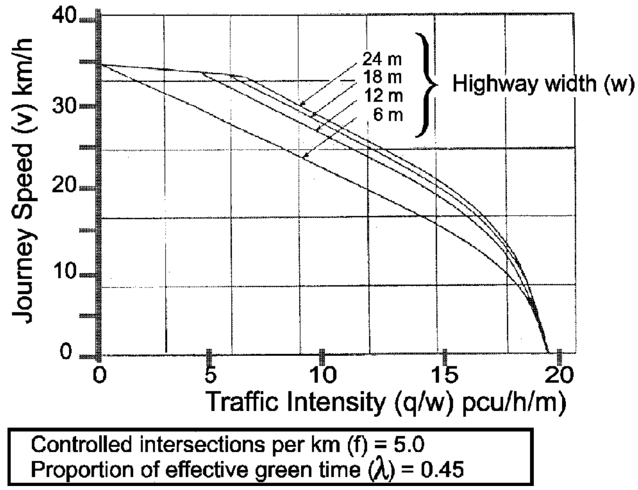


Figure 6.11
Effect of Roadway Width on Relation Between Average (Journey) Speed and Flow in Typical Case (Wardrop 1968, Figure 5).

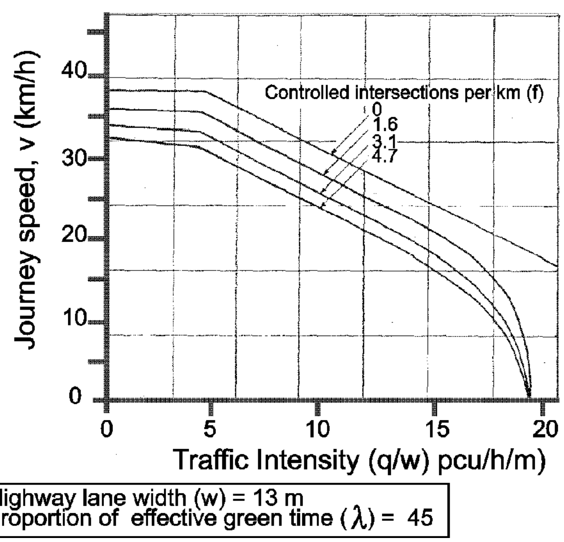
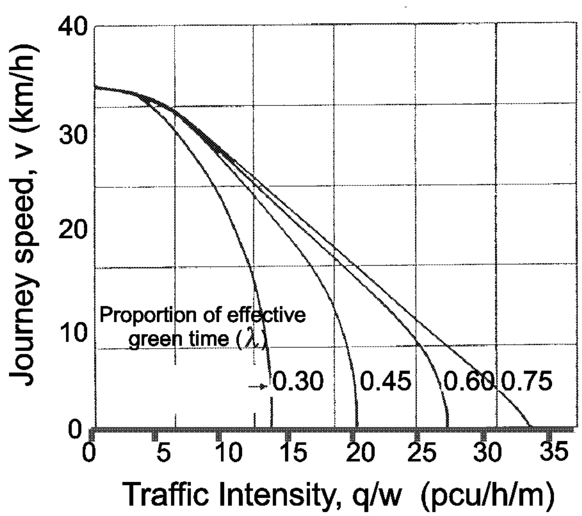


Figure 6.12
Effect of Number of Intersections Per Mile on Relation Between Average (Journey) Speed and Flow in Typical Case (Wardrop 1968, Figure 6).



Highway width (w) = 12m
Controlled intersections per km (f) = 5.0

Figure 6.13
Effect of Capacity of Intersections on Relation Between Average (Journey) Speed and Flow in Typical Case (Wardrop 1968, Figure 7).

Godfrey (1969) examined the relations between the average speed and the concentration (defined as the number of vehicles in the network), shown in Figure 6.14, and between average speed and the vehicle miles traveled in the network in one hour, shown in Figure 6.15. Floating vehicles on circuits within the network were used to estimate average speed and aerial photographs were used to estimate concentration.

There is a certain concentration that results in the maximum flow (or the maximum number of miles traveled, see Figure 6.15), which occurs around 10 miles/hour. As traffic builds up past this optimum, average speeds show little deterioration, but there is excessive queuing to get into the network (either from car parking lots within the network or on streets leading into the designated network). Godfrey also notes that expanding an intersection to accommodate more traffic will move the queue to another location within the network, unless the bottlenecks downstream are cleared.

6.2.3 General Network Models Incorporating Network Parameters

Some models have defined specific parameters which intend to quantify the quality of traffic service provided to the users in the network. Two principal models are discussed in this chapter, the α-relationship, below, and the two-fluid theory of town traffic. The two-fluid theory has been developed and applied to a greater extent than the other models discussed in this section, and is described in Section 6.3.

Zahavi (1972a; 1972b) selected three principal variables, I, the traffic intensity (here defined as the distance traveled per unit area), R, the road density (the length or area of roads per unit area), and v, the weighted space mean speed. Using data from England and the United States, values of I, v, and R were found for different regions in different cities. In investigating various relationships between I and v/R, a linear fit was found between the logarithms of the variables:

$$I = \alpha(v/R)^m \quad , \tag{6.27}$$

where α and m are parameters. Trends for London and Pittsburgh are shown in Figure 6.16. The slope (m) was found to be close to -1 for all six cities examined, reducing Equation 6.27 to

$$I = \alpha R/v \quad , \tag{6.28}$$

where α is different for each city. Relative values of the variables were calculated by finding the ratio between observed

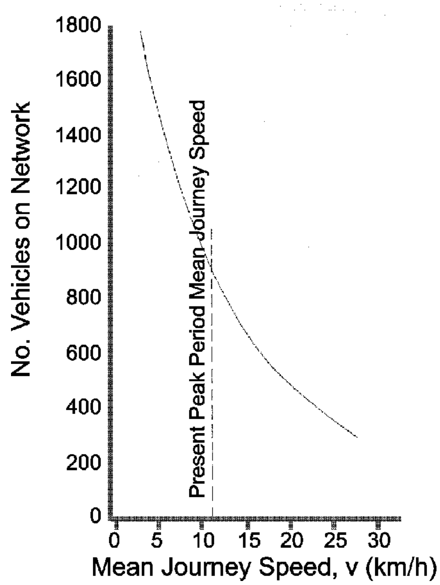


Figure 6.14
Relationship Between Average (Journey) Speed and Number of Vehicles on Town Center Network (Godfrey 1969, Figure 1).

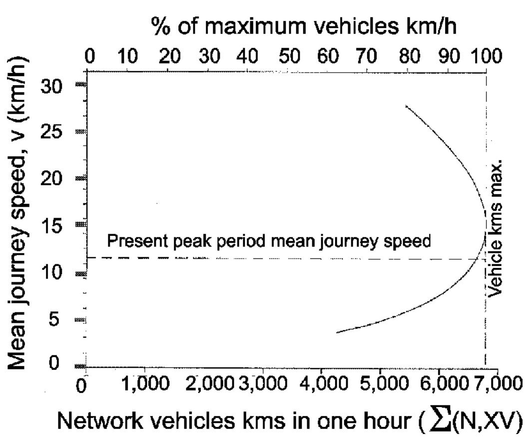


Figure 6.15
Relationship Between Average (Journey) Speed of Vehicles and Total Vehicle Mileage on Network (Godfrey 1969, Figure 2).

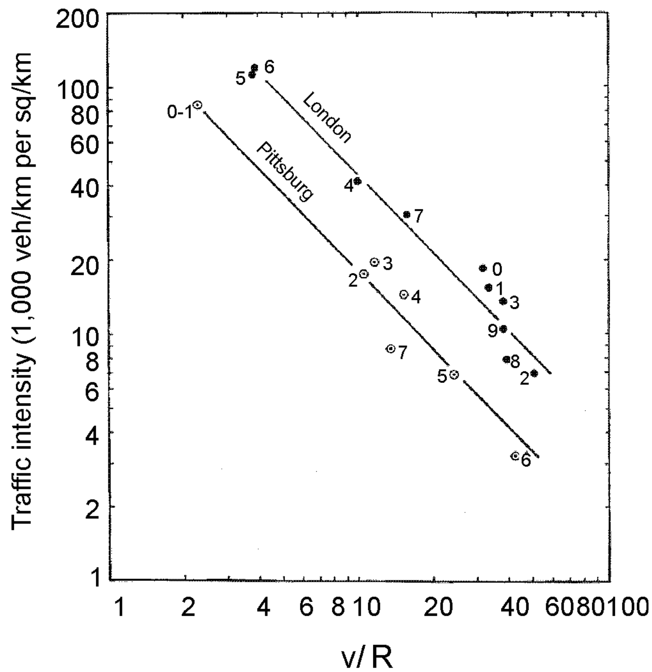


Figure 6.16
The α-Relationship for the Arterial Networks of London and Pittsburgh, in Absolute Values (Zahavi 1972a, Figure 1).

values of *I* and *v/R* for each sector and the average value for the entire city. The relationship between the relative values is shown in Figure 6.17, where the observations for London and Pittsburgh fall along the same line.

The physical characteristics of the road network, such as street widths and intersection density, were found to have a strong effect on the value of α for each zone in a city. Thus, α may serve as a measure of the combined effects of the network characteristics and traffic performance, and can possibly be used as an indicator for the level of service. The α map of London is shown in Figure 6.18. Zones are shown by the dashed lines, with dotted circles indicating zone centroids. Values of α were calculated for each zone and contour lines of equal α were drawn, showing areas of (relatively) good and poor traffic flow conditions. (The quality of traffic service improves with increasing α.)

Unfortunately, Buckley and Wardrop (1980) have shown that α is strongly related to the space mean speed, and Ardekani (1984), through the use of aerial photographs, has shown that α has a high positive correlation with the network concentration.

The two-fluid model also uses parameters to evaluate the level of service in a network and is described in Section 6.3.

6.2.4 Continuum Models

Models have been developed which assume an arbitrarily fine grid of streets, i.e., infinitely many streets, to circumvent the errors created on the relatively sparse networks typically used during the trip or network assignment phase in transportation planning (Newell 1980). A basic street pattern is superimposed over this continuum of streets to restrict travel to appropriate directions. Thus, if a square grid were used, travel on the street network would be limited to the two available directions (the x and y directions in a Cartesian plot), but origins and destinations could be located anywhere in the network.

Individual street characteristics do not have to be specifically modeled, but network-wide travel time averages and capacities (per unit area) must be used for traffic on the local streets. Other street patterns include radial-ring and other grids (triangular, for example).

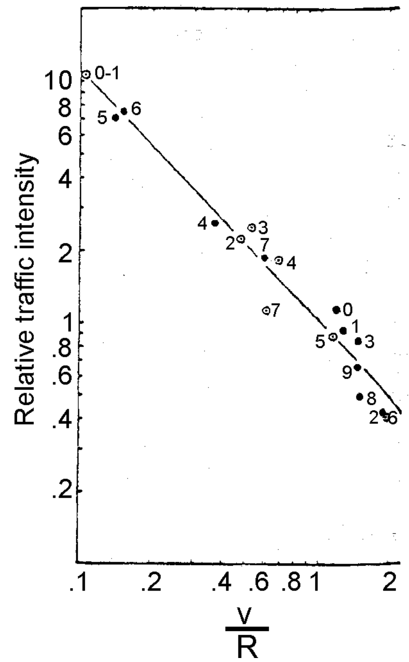


Figure 6.17
The α-Relationship for the Arterial Networks of London and Pittsburgh, in Relative Values (Zahavi 1972a, Figure 2).

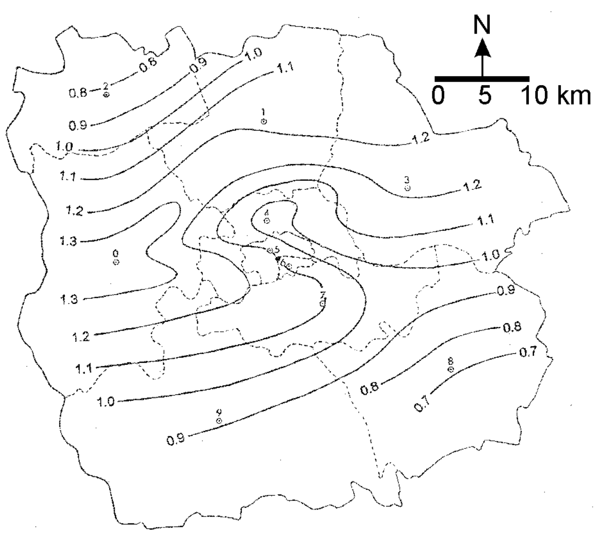


Figure 6.18
The α-Map for London, in Relative Values (Zahavi 1972b, Figure 1).

While the continuum comprises the local streets, the major streets (such as arterials and freeways) are modeled directly. Thus, the continuum of local streets provides direct access (within the constraints provided by the superimposed grid) to the network of major streets.

6.3 Two-Fluid Theory

An important result from Prigogine and Herman's (1971) kinetic theory of traffic flow is that two distinct flow regimes can be shown. These are individual and collective flows and are a function of the vehicle concentration. When the concentration rises so that the traffic is in the collective flow regime, the flow pattern becomes largely independent of the will of individual drivers.

Because the kinetic theory deals with multi-lane traffic, the two-fluid theory of town traffic was proposed by Herman and Prigogine (Herman and Prigogine 1979; Herman and Ardekani 1984) as a description of traffic in the collective flow regime in an urban street network. Vehicles in the traffic stream are divided into two classes (thus, two fluid): moving and stopped vehicles. Those in the latter class include vehicles stopped in the traffic stream, i.e., stopped for traffic signals and stop signs, stopped for vehicles loading and unloading which are blocking a moving lane, stopped for normal congestion, etc., but excludes those out of the traffic stream (e.g., parked cars).

The two-fluid model provides a macroscopic measure of the quality of traffic service in a street network which is independent of concentration. The model is based on two assumptions:

(1) The average running speed in a street network is proportional to the fraction of vehicles that are moving, and
(2) The fractional stop time of a test vehicle circulating in a network is equal to the average fraction of the vehicles stopped during the same period.

The variables used in the two-fluid model represent network-wide averages taken over a given period of time.

The first assumption of the two-fluid theory relates the average speed of the moving (running) vehicles, V_r, to the fraction of moving vehicles, f_r, in the following manner:

$$V_r = V_m f_r^{\,n} , \qquad (6.29)$$

where V_m and n are parameters. V_m is the average maximum running speed, and n is an indicator of the quality of traffic service in the network; both are discussed below. The average speed, V, can be defined as $V_r f_r$, and combining with Equation 6.29,

$$V = V_m f_r^{\,n+1} . \qquad (6.30)$$

Since $f_r + f_s = 1$, where f_s is the fraction of vehicles stopped, Equation 6.30 can be rewritten

$$V = V_m (1 - f_s)^{n+1} . \qquad (6.31)$$

Boundary conditions are satisfied with this relation: when f_s=0, V=V_m, and when f_s=1, V=0.

This relation can also be expressed in average travel times rather than average speeds. Note that T represents the average travel time, T_r the running (moving) time, and T_s the stop time, all per unit distance, and that $T=1/V$, $T_r=1/V_r$, and $T_m=1/V_m$, where T_m is the average minimum trip time per unit distance.

The second assumption of the two-fluid model relates the fraction of time a test vehicle circulating in a network is stopped to the average fraction of vehicles stopped during the same period, or

$$f_s = \frac{T_s}{T} . \qquad (6.32)$$

This relation has been proven analytically (Ardekani and Herman 1987), and represents the ergodic principle embedded in the model, i.e., that the network conditions can be represented by a single vehicle appropriately sampling the network.

Restating Equation 6.31 in terms of travel time,

$$T = T_m \left(1 - f_s\right)^{-(n+1)} . \qquad (6.33)$$

Incorporating Equation 6.32,

$$T = T_m \left[1 - (T_s / T)\right]^{-(n+1)} , \qquad (6.34)$$

realizing that $T = T_r + T_s$, and solving for T_r,

$$T_r = T_m^{\frac{1}{n+1}} \; T^{\frac{n}{n+1}} . \qquad (6.35)$$

The formal two-fluid model formulation, then, is

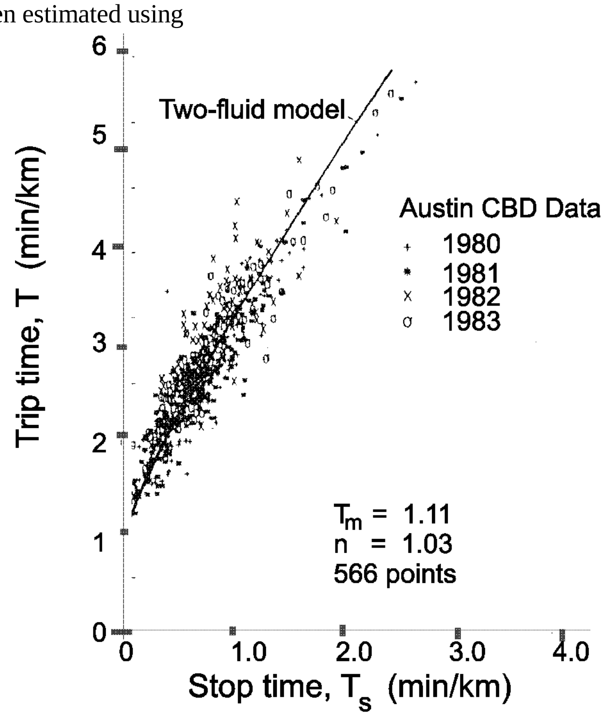
$$T_s = T - T_m^{\frac{1}{n+1}} \; T^{\frac{n}{n+1}} . \qquad (6.36)$$

A number of field studies have borne out the two-fluid model (Herman and Ardekani 1984; Ardekani and Herman 1987; Ardekani et al. 1985); and have indicated that urban street networks can be characterized by the two model parameters, n and T_m. These parameters have been estimated using observations of stopped and moving times gathered in each network. The log transform of Equation 6.35,

$$ln\ T_r = \frac{1}{n+1}\, ln\ T_m + \frac{n}{n+1}\, ln\ T \qquad (6.37)$$

provides a linear expression for the use of least squares analysis.

Empirical information has been collected with chase cars following randomly selected cars in designated networks. Runs have been broken into one- or two-mile trips, and the running time *(T_r)* and total trip time *(T)* for each one- or two-mile trip from the observations for the parameter estimation. Results tend to form a nearly linear relationship when trip time is plotted against stop time (Equation 6.36) as shown in Figure 6.19 for data collected in Austin, Texas. The value of T_m is reflected by the y-intercept (i.e., T at T_s=0), and n by the slope of the curve. Data points representing higher concentration levels lie higher along the curve.



Note: Each point represents one test run approximately 1 or 2 miles long.

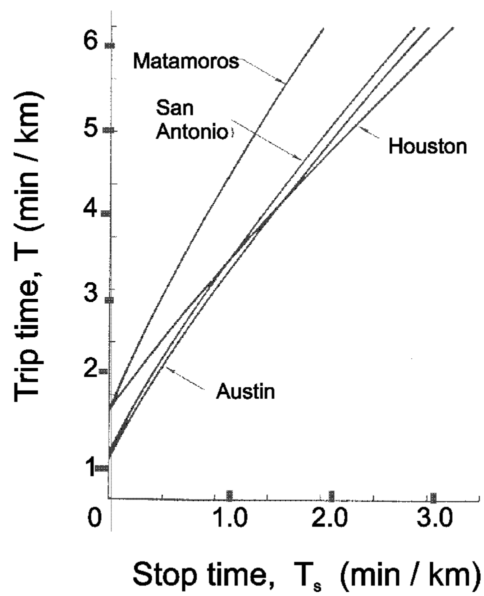
Figure 6.19
Trip Time vs. Stop Time for the Non-Freeway Street Network of the Austin CBD (Herman and Ardekani 1984, Figure 3).

6.3.1 Two-Fluid Parameters

The parameter T_m is the average minimum trip time per unit distance, and it represents the trip time that might be experienced by an individual vehicle alone in the network with no stops. This parameter is unlikely to be measured directly, since a lone vehicle driving though the network very late at night is likely to have to stop at a red traffic signal or a stop sign. T_m, then, is a measure of the uncongested speed, and a higher value would indicate a lower speed, typically resulting in poorer operation. T_m has been found to range from 1.5 to 3.0 minutes/mile, with smaller values typically representing better operating conditions in the network.

As stop time per unit distance (T_s) increases for a single value of n, the total trip time also increases. Because $T=T_r+T_s$, the total trip time must increase at least as fast as the stop time. If n=0, T_r is constant (by Equation 6.35), and trip time would increase at the same rate as the stop time. If n>0, trip time increases at a faster rate than the stop time, meaning that running time is also increasing. Intuitively, n must be greater than zero, since the usual cause for increased stop time is increased congestion, and when congestion is high, vehicles when moving, travel at a lower speed (or higher running time per unit distance) than they do when congestion is low. In fact, field studies have shown that n varies from 0.8 to 3.0, with a smaller value typically indicating better operating conditions in the network. In other words, n is a measure of the resistance of the network to degraded operation with increased demand. Higher values of n indicate networks that degrade faster as demand increases. Because the two-fluid parameters reflect how the network responds to changes in demand, they must be measured and evaluated in a network over the entire range of demand conditions.

While lower n and T_m values represent, in general, better traffic operations in a network, often there is a tradeoff. For example, two-fluid trends for four cities are shown in Figure 6.20. In comparing Houston (T_m=2.70 min/mile, n=0.80) and Austin (T_m=1.78 min/mile, n=1.65), one finds that traffic in Austin moves at significantly higher average speeds during off-peak conditions (lower concentration); at higher concentrations, the curves essentially overlap, indicating similar operating conditions. Thus, despite a higher value of n, traffic conditions



Note: Trip Time vs. Stop Time Two-Fluid Model Trends for CBD Data From the Cities of Austin, Houston, and San Antonio, Texas, and Matamoros, Mexico.

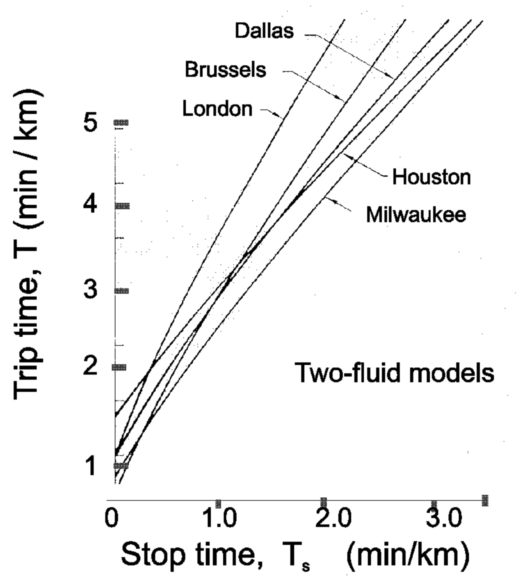
Figure 6.20
Trip Time vs. Stop Time Two-Fluid Model Trends
(Herman and Ardekani 1984, Figure 6).

are better in Austin than Houston, at least at lower concentrations. Different values of the two-fluid parameters are found for different city street networks, as was shown above and in Figure 6.21. The identification of specific features which have the greatest effect on these parameters has been approached through extensive field studies and computer simulation.

6.3.2 Two-Fluid Parameters: Influence of Driver Behavior

Data for the estimation of the two-fluid parameters is collected through chase car studies, where the driver is instructed to follow a randomly selected vehicle until it either parks or leaves the designated network, after which a nearby vehicle is selected and followed. The chase car driver is instructed to follow the vehicle being chased imitating the other driver's actions so as to reflect, as closely as possible, the fraction of time the other driver spends stopped. The objective is to sample the behavior of the drivers in the network as well as the commonly used routes in the street network. The chase car's trip history is then broken into one-mile (typically) segments, and T_r and T calculated for each mile. The (T_r, T) observations are then used in the estimation of the two-fluid parameters.

One important aspect of the chase car study is driver behavior, both that of the test car driver and the drivers sampled in the network. One study addressed the question of extreme driver behaviors, and found that a test car driver instructed to drive aggressively established a significantly different two-fluid trend than one instructed to drive conservatively in the same network at the same time (Herman et al. 1988).

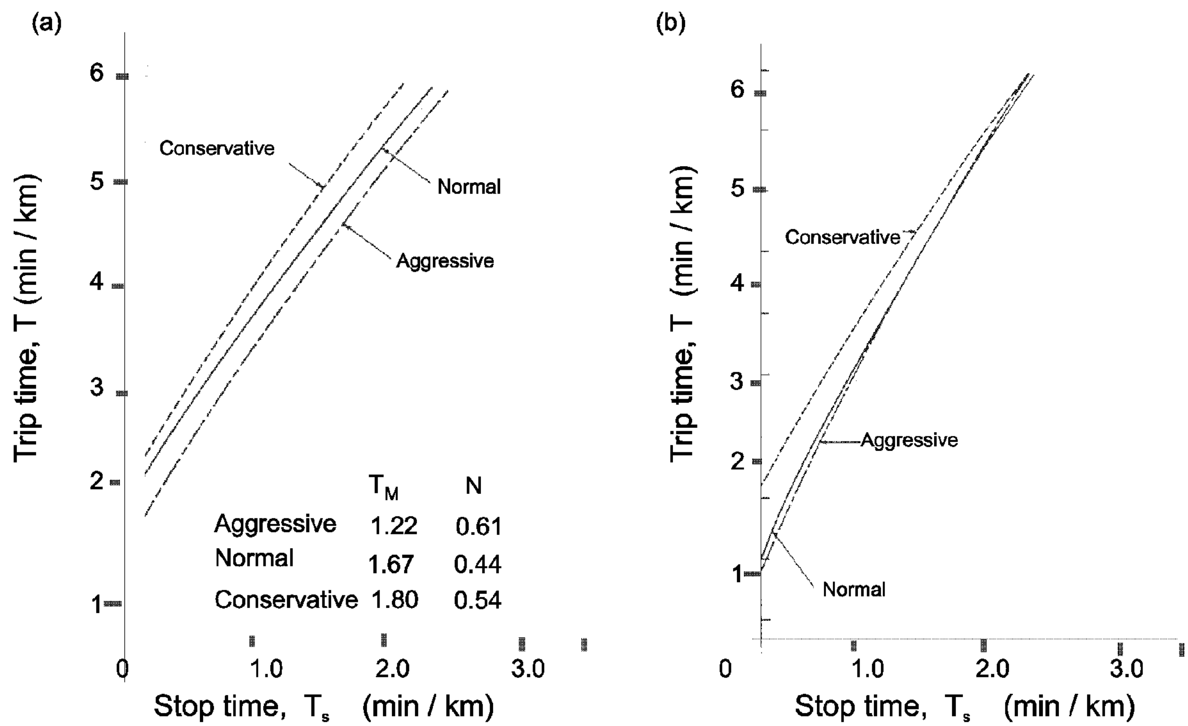


Note: Trip Time vs. Stop Time Two-Fluid Model Trends for Dallas and Houston, Texas, compared to the trends in Milwaukee, Wisconsin, and in London and Brussels.

Figure 6.21
Trip Time vs. Stop Time Two-Fluid Model Trends Comparison
(Herman and Ardekani 1984, Figure 7).

The two-fluid trends resulting from the these studies in two cities are shown in Figure 6.22. In both cases, the normal trend was found through a standard chase car study, conducted at the same time as the aggressive and conservative test drivers were in the network. In both cases, the two-fluid trends established by the aggressive and conservative driver are significantly different. In Roanoke (Figure 6.22a), the normal trend lies between the aggressive and conservative trends, as expected. However, the aggressive trend approaches the normal trend at high demand levels, reflecting the inability of the aggressive driver to reduce his trip and stop times during peak periods. On the other hand, at lower network concentrations, the aggressive driver can take advantage of the less crowded streets and significantly lower his trip times.

As shown in Figure 6.22b, aggressive driving behavior more closely reflects normal driving habits in Austin, suggesting more aggressive driving overall. Also, all three trends converge at high demand (concentration) levels, indicating that, perhaps, the Austin network would suffer congestion to a greater extent than Roanoke, reducing all drivers to conservative behavior (at least as represented in the two-fluid parameters).



Note: The two-fluid trends for aggressive, normal, and conservative drivers in (a) Roanoke, Virginia, and (b) Austin, Texas

Figure 6.22
Two-Fluid Trends for Aggressive, Normal, and Conservative Drivers (Herman et al. 1988, Figures 5 and 8).

The results of this study reveal the importance of the behavior of the chase car driver in standard two-fluid studies. While the effects on the two-fluid parameters of using two different chase car drivers in the same network at the same time has not been investigated, there is thought to be little difference between two well-trained drivers. To the extent possible, however, the same driver has been used in different studies that are directly compared.

6.3.3 Two-Fluid Parameters: Influence of Network Features (Field Studies)

Geometric and traffic control features of a street network also play an important role in the quality of service provided by a network. If relationships between specific features and the two-fluid parameters can be established, the information could be used to identify specific measures to improve traffic flow and provide a means to compare the relative improvements.

Ayadh (1986) selected seven network features: lane miles per square mile, number of intersections per square mile, fraction of one-way streets, average signal cycle length, average block length, average number of lanes per street, and average block length to block width ratio. The area of the street network under consideration is used with the first two variables to allow a direct comparison between cities. Data for the seven variables were collected for four cities from maps and in the field. Through a regression analysis, the following models were selected:

$$\begin{aligned} T_m &= 3.59 - 0.54\,C_6 \quad \text{and} \\ n &= -0.21 + 2.97\,C_3 + 0.22\,C_7 \end{aligned} \tag{6.38}$$

where C_3 is the fraction of one-way streets, C_6 the average number of lanes per street, and C_7 the average block length to block width ratio. Of these network features, only one (the fraction of one-way streets) is relatively inexpensive to implement. One feature, the block length to block width ratio, is a topological feature which would be considered fixed for any established street network.

Ardekani et al. (1992), selected ten network features: average block length, fraction of one-way streets, average number of lanes per street, intersection density, signal density, average speed limit, average cycle length, fraction of curb miles with parking allowed, fraction of signals actuated, and fraction of approaches with signal progression. Of these, only two features (average block length and intersection density) can be considered fixed, and, as such, not useful in formulating network improvements. In addition, one feature (average number of lanes per street), also used in the previous study (Ayadh 1986), can typically be increased only by eliminating parking (if present), yielding only limited opportunities for improvement of traffic flow. Data was collected in ten cities; in seven of the cities, more than one study was conducted as major geometric changes or revised signal timings were implemented, yielding nineteen networks for this study. As before, the two-fluid parameters in each network were estimated from chase car data and the network features were determined from maps, field studies, and local traffic engineers. Regression analysis yielded the following models:

$$\begin{aligned} T_m &= 3.93 + 0.0035\,X_5 - 0.047\,X_6 - 0.433\,X_{10} \\ \text{and}\ \ n &= 1.73 + 1.124\,X_2 - 0.180\,X_3 - 0.0042\,X_5 - 0.271\,X_9 \end{aligned} \tag{6.39}$$

where X_2 is the fraction of one-way streets, X_3 the average number of lanes per street, X_5 the signal density, X_6 the average speed limit, X_9 the fraction of actuated signals, and X_{10} the fraction of approaches with good progression. The R^2 for these equations, 0.72 and 0.75 (respectively), are lower than those for Equation 6.38 (both very close to 1), reflecting the larger data size. The only feature in common with the previous model (Equation 6.38) is the appearance of the fraction of one-way streets in the model for *n*. Since all features selected can be changed through operational practices (signal density can be changed by placing signals on flash), the models have potential practical application. Computer simulation has also been used to investigate these relationships, and is discussed in Section 6.3.4.

6.3.4 Two-Fluid Parameters: Estimation by Computer Simulation

Computer simulation has many advantages over field data in the study of network models. Conditions not found in the field can be evaluated and new control strategies can be easily tested. In the case of the two-fluid model, the entire vehicle population in the network can be used in the estimation of the model parameters, rather than the small sample used in the chase car studies. TRAF-NETSIM (Mahmassani et al. 1984), a microscopic traffic simulation model, has been used successfully with the two-fluid model.

Most of the simulation work to-date has used a generic grid network in order to isolate the effects of specific network features on the two-fluid parameters (FHWA 1993). Typically, the simulated network has been a 5 x 5 intersection grid made up entire of two-way streets. Traffic signals are at each intersection and uniform turning movements are applied throughout. The network is closed, i.e., vehicles are not allowed to leave the network, thus maintaining constant concentration during the simulation run. The trip histories of all the vehicles circulating in the network are aggregated to form a single $(T_r\ ,\ T)$ observation for use in the two-fluid parameter estimation. A series of five to ten runs over a range of network concentrations (nearly zero to 60 or 80 vehicles/lane-mile) are required to estimate the two-fluid parameters.

Initial simulation runs in the test network showed both T and T_s increasing with concentration, but T_r remaining nearly constant, indicating a very low value of n (Mahmassani et al. 1984). In its default condition, NETSIM generates few of the vehicle interaction of the type found in most urban street networks, resulting in flow which is much more idealized than in the field. The short-term event feature of NETSIM was used to increase the inter-vehicular interaction (Williams et al. 1985). With this feature, NETSIM blocks the right lane of the specified link at mid-block; the user specifies the average time for each blockage and the number of blockages per hour, which are stochastically applied by NETSIM. In effect, this represents a vehicle stopping for a short time (e.g., a commercial vehicle unloading goods), blocking the right lane, and requiring vehicles to change lanes to go around it. The two-fluid parameters (and n in particular) were very sensitive to the duration and frequency of the short-term events. For example, using an average 45-second event every two minutes, n rose from 0.076 to 0.845 and T_m fell from 2.238 to 2.135. With the use of the short-term events, the values of both parameters were within the ranges found in the field studies. Further simulation studies found both block length (here, distance between signalized intersections) and the use of progression to have significant effects on the two-fluid parameters (Williams et al. 1985).

Simulation has also provided the means to investigate the use of the chase car technique in estimating the two-fluid parameters (Williams et al. 1995). The network-wide averages in a simulation model can be directly computed; and chase car data can be simulated by recording the trip history of a single vehicle for one mile, then randomly selecting another vehicle in the network. Because the two-fluid model is non-linear (specifically, Equation 6.35, the log transform of which is used to estimate the parameters), estimations performed at the network level and at the individual vehicle level result in different values of the parameters, and are not directly comparable. The sampling strategy, which was found to provide the best parameter estimates, required a single vehicle circulating in the network for at least 15 minutes. However, due to the wide variance of the estimate (due to the possibility of a relatively small number of "chased" cars dominating the sample estimation), the estimate using a single vehicle was often far from the parameter estimated at the network level. On the other hand, using 20 vehicles to sample the network resulted in estimates much closer to those at the network level. The much smaller variance of the estimates made with twenty vehicles, however, resulted in the estimate being significantly different from the network-level estimate. The implication of this study is that, while estimates at the network and individual vehicle levels can not be directly compared, as long as the same sampling strategy is used, the resulting two-fluid parameters, although biased from the "true" value, can be used in making direct comparisons.

6.3.5 Two-Fluid Parameters: Influence of Network Features (Simulation Studies)

The question in Section 6.3.3, above, regarding the influence of geometric and control features of a network on the two-fluid parameters was revisited with an extensive simulation study (Bhat 1994). The network features selected were: average block length, fraction of one-way streets, average number of lanes per street, signals per intersection, average speed limit, average signal cycle length, fraction of curb miles with parking, and fraction of signalized approaches in progression. A uniform-precision central composite design was selected as the experimental design, resulting in 164 combination of the eight network variables. The simulated network was increased to 11 by 11 intersections; again, vehicles were not allowed to leave the network, but traffic data was collected only on the interior 9 by 9 intersection grid, thus eliminating the edge effects caused by the necessarily different turning movements at the boundaries. Ten simulation runs were made for each combination of variables over a range of concentrations from near zero to about 35 vehicles/lane-mile.

Regression analysis yielded the following models:

$$T_m = 1.049 + 1.453\,X_2 + 0.684\,X_3 - 0.024\,X_6 \quad \text{and} \\ n = 4.468 - 1.391\,X_3 - 0.048\,X_5 + 0.042\,X_6 \qquad (6.40)$$

where X_2 is the fraction of one-way streets, X_3 the number of lanes per street, X_5 average speed limit, and X_6 average cycle length. The R^2 (0.26 and 0.16 for Equation 6.40) was considerably lower than that for the models estimated with data from field studies (Equation 6.39). Additionally, the only variable in common between Equations 6.39 and 6.40 is the number of lanes per street in the equation for n. Additional work is required to clarify these relationships.

6.3.6 Two-Fluid Model: A Practical Application

When the traffic signals in downtown San Antonio were retimed, TRAF-NETSIM was selected to quantify the improvements in the network. In order to assure that the results reported by NETSIM reflected traffic conditions in San Antonio, NETSIM was calibrated with the two-fluid model.

Turning movement counts used in the development of the new signal timing plans were available for coding NETSIM. Simulation runs were made for 31 periods throughout the day, and the two-fluid parameters were estimated and compared with those found in the field. By a trial and error process, NETSIM was calibrated by

- Increasing the sluggishness of drivers, by increasing headways during queue discharge at traffic signals and reducing maximum acceleration,
- Adding vehicle/driver types to increase the range of sluggishness represented in the network, and
- Reducing the desired speed on all links to 32.2 km/h during peaks and 40.25 km/h otherwise (Denney 1993).

Three measures of effectiveness (MOEs) were used in the evaluation: total delay, number of stops, and fuel consumption. The changes noted for all three MOEs were greater between calibrated and uncalibrated NETSIM results than between before and after results. Reported relative improvements were also affected. The errors in the reported improvements without calibration ranged from 16 percent to 132 percent (Denney 1994).

6.4 Two-Fluid Model and Traffic Network Flow Models

Computer simulation provides an opportunity to investigate network-level relationships between the three fundamental variables of traffic flow, speed *(V)*, flow *(Q)*, and concentration *(K)*, defined as average quantities taken over all vehicles in the network over some observation period (Mahmassani et al. 1984). While the existence of "nice" relations between these variables could not be expected, given the complexity of network interconnections, simulation results indicate relationships similar to those developed for arterials may be appropriate (Mahmassani et al. 1984; Williams et al. 1985). A series of simulation runs, as described in Section 6.3.4, above, was made at concentration levels between 10 and 100 vehicles/lane-mile. The results are shown in Figure 6.23, and bear a close resemblance to their counterparts for individual road sections. The fourth plot shows the relation of f_s, the fraction of vehicles stopped from the two-fluid model, to the concentration. In addition, using values of flow, speed, and concentration independently computed from the simulations, the network-level version of the fundamental relation $Q=KV$ was numerically verified (Mahmassani et al. 1984; Williams et al. 1987).

Three model systems were derived and tested against simulation results (Williams et al. 1987; Mahmassani et al. 1987); each model system assumed $Q=KV$ and the two-fluid model, and consisted of three relations:

$$V = f(K)\ , \qquad (6.41)$$

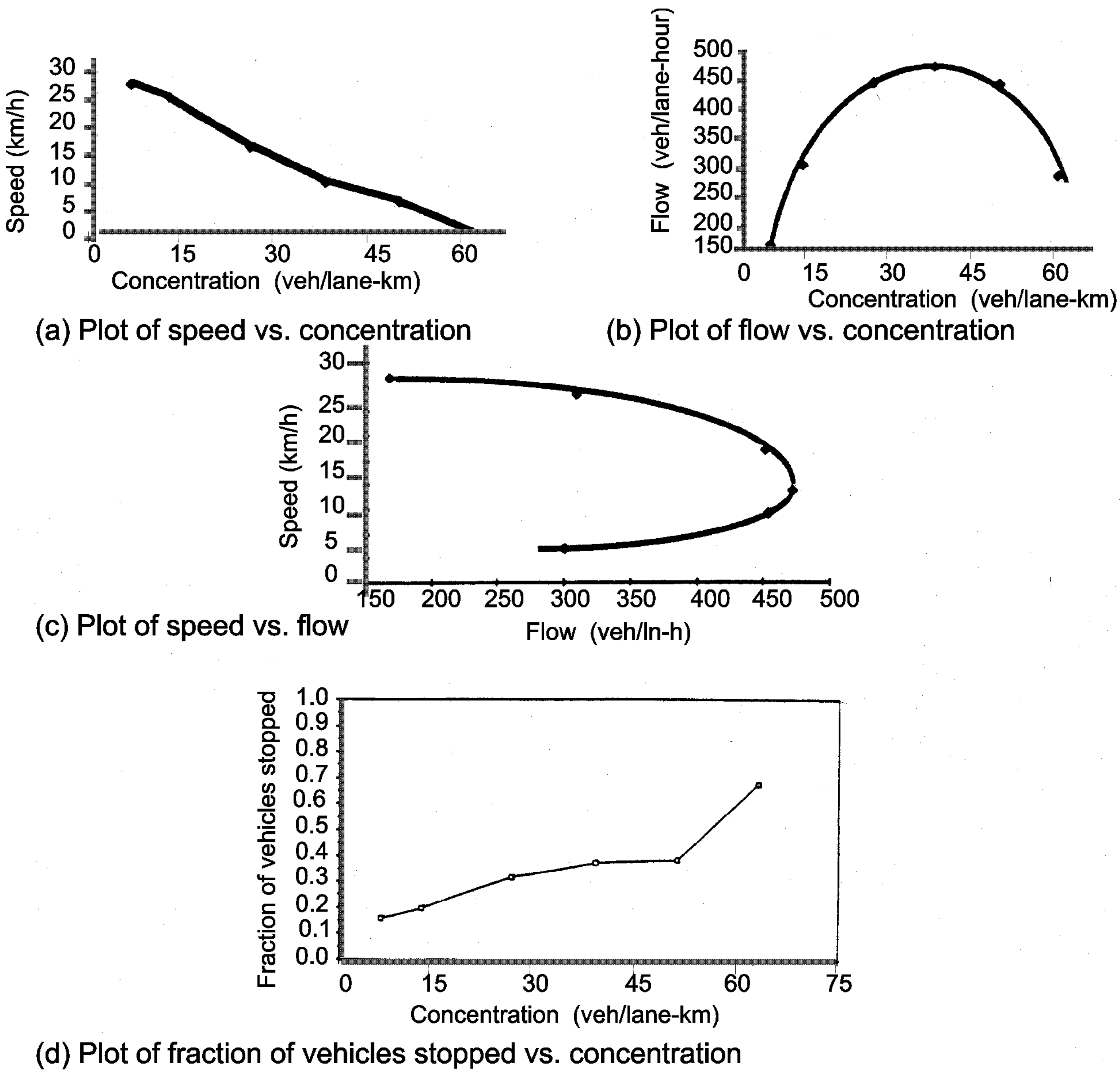


Figure 6.23
Simulation Results in a Closed CBD-Type Street Network.
(Williams et al. 1987, Figures 1-4).

A model system is defined by specifying one of the above

$$Q = g(K) \quad , \text{ and} \tag{6.42}$$

$$f_s = h(K) \quad . \tag{6.43}$$

relationships; the other two can then be analytically derived. (A relation between Q and V could also be derived.)

Model System 1 is based on a postulated relationship between the average fraction of vehicles stopped and the network concentration from the two-fluid theory (Herman and Prigogine

1979), later modified to reflect that the minimum $f_s > 0$ (Ardekani and Herman 1987):

$$f_s = f_{s,\min} + (1 - f_{s,\min})\,(K/K_j)^{\pi} \quad , \qquad (6.44)$$

where $f_{s,min}$ is the minimum fraction of vehicles stopped in a network, K_j is the jam concentration (at which the network is effectively saturated), and π is a parameter which reflects the quality of service in a network. The other two relations can be readily found, first by substituting f_s from Equation 6.44 into Equation 6.31:

$$V = V_m\,(1 - f_{s,\min})^{n+1}\,[1 - (K/K_j)^{\pi}]^{n+1} \quad , \qquad (6.45)$$

then by using $Q=KV$,

$$Q = K\,V_m\,(1 - f_{s,\min})^{n+1}\,[1 - (K/K_j)^{\pi}]^{n+1} \quad . \qquad (6.46)$$

Equations 6.44 through 6.46 were fitted to the simulated data and are shown in Figure 6.24. Because the point representing the highest concentration (about 100 vehicles/lane-mile) did not lie in the same linear $\ln T_r$ - $\ln T$ trend as the other points, the two-fluid parameters n and T_m were estimated with and without the highest concentration point, resulting in the Method 1 and Method 2 curves, respectively, in the V-K and Q-K curves in Figure 6.24.

Model System 2 adopts Greenshields' linear speed-concentration relationship (Gerlough and Huber 1975),

$$V = V_f\,(1 - K/K_j) \quad , \qquad (6.47)$$

where V_f is the free flow speed (and is distinct from V_m ; $V_f \le V_m$ always, and typically $V_f < V_m$). The f_s-K relation can be found by substituting Equation 6.47 into Equation 6.31 and solving for f_s:

$$f_s = 1 - [(V_f/V_m)\,(1 - K/K_j)]^{1/(n+1)} \quad , \qquad (6.48)$$

then by using $Q=KV$,

$$Q = V_f\,(K - K^2/K_j) \quad . \qquad (6.49)$$

Equations 6.47 through 6.49 were fitted to the simulation data and are shown in Figure 6.25. The difference between the Method 1 and Method 2 curves in the f_s-K plot (Figure 6.25) is described above. Model System 3 uses a non-linear bell-shaped function for the V-K model, originally proposed by Drake, et al., for arterials (Gerlough and Huber 1975):

$$V = V_f\,\exp[-\alpha\,(K/K_m)^d\,] \quad , \qquad (6.50)$$

where K_m is the concentration at maximum flow, and α and d are parameters. The f_s-K and Q-K relations can be derived as shown for Model System 2:

$$f_s = 1 - \{(V_f/V_m)\,\exp[-\alpha\,(K/K_m)^d\,]\}^{1/(n+1)} \quad \text{and} \qquad (6.51)$$

$$Q = K\,V_f\,\exp[-\alpha\,(K/K_m)^d\,] \quad . \qquad (6.52)$$

Equations 6.50 through 6.52 were fitted to the simulation data and are shown in Figure 6.26.

Two important conclusions can be drawn from this work. First, that relatively simple macroscopic relations between network-level variables appear to work. Further, two of the models shown are similar to those established at the individual facility level. Second, the two-fluid model serves well as the theoretical link between the postulated and derived functions, providing another demonstration of the model's validity. In the second and third model systems particularly, the derived f_s-K function performed remarkable well against the simulated data, even though it was not directly calibrated using that data.

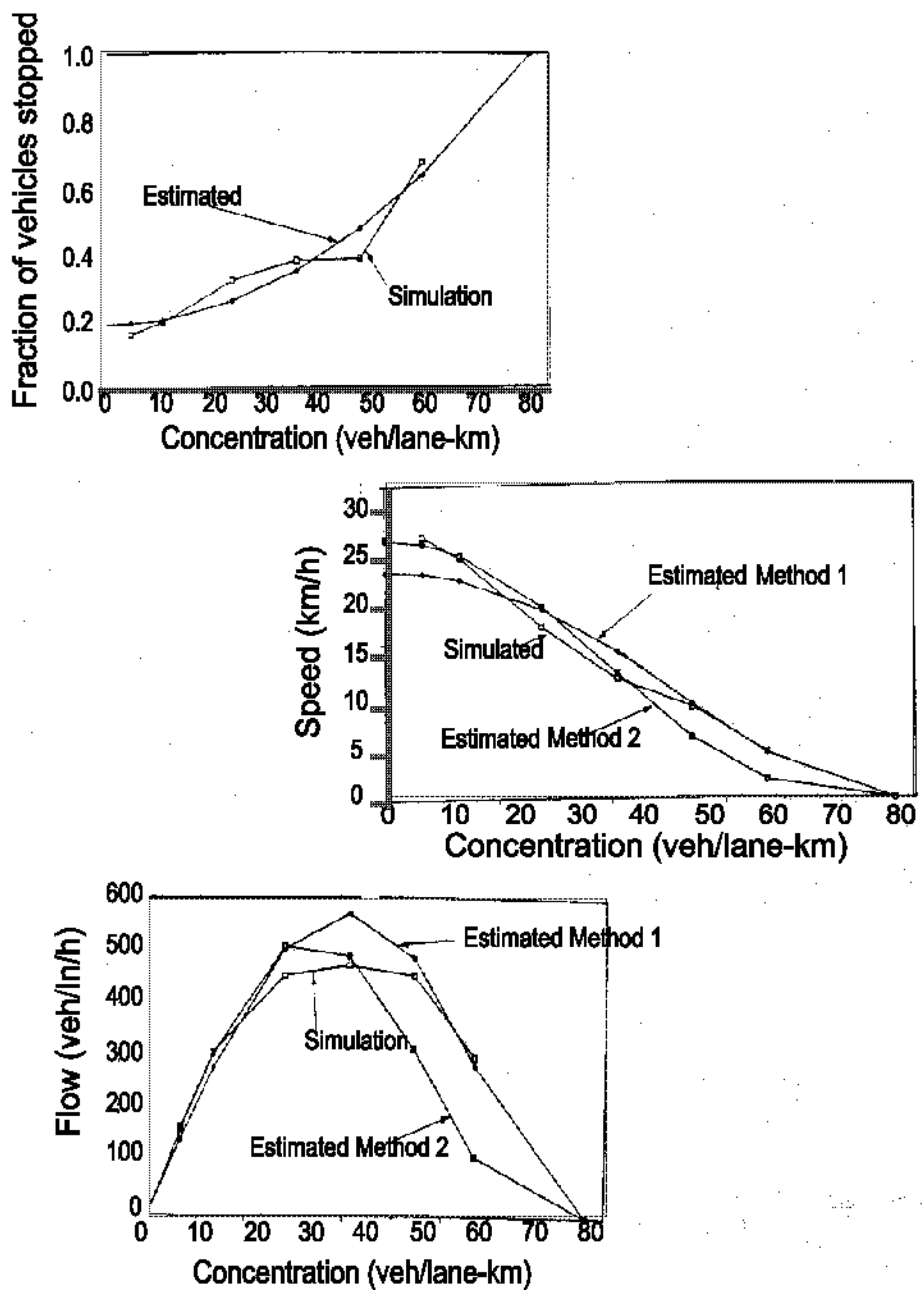


Figure 6.24
Comparison of Model System 1 with Observed Simulation Results
(Williams et al. 1987, Figure 5, 7, and 8).

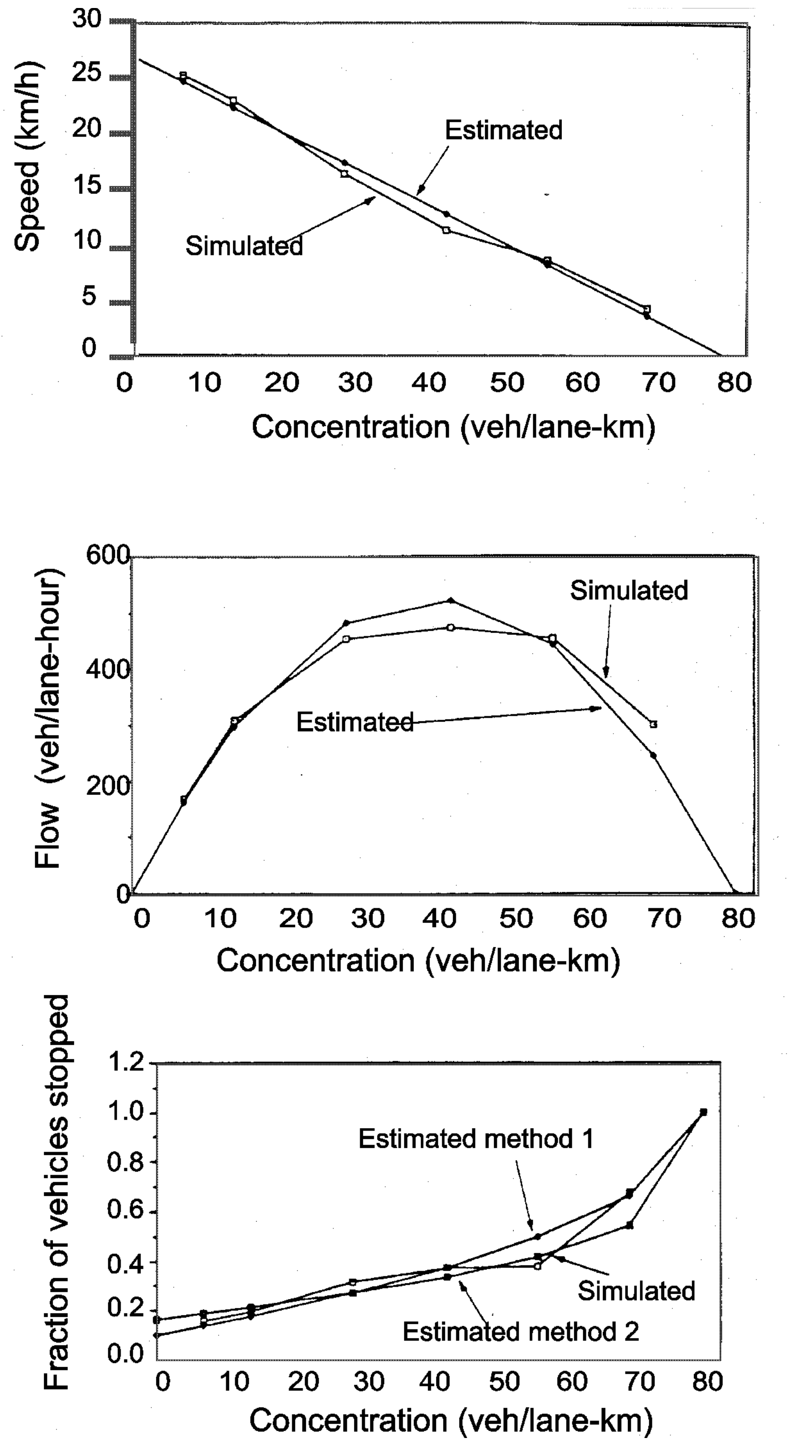


Figure 6.25
Comparison of Model System 2 with Observed Simulation Results
(Williams et al. 1987, Figures 9-11).

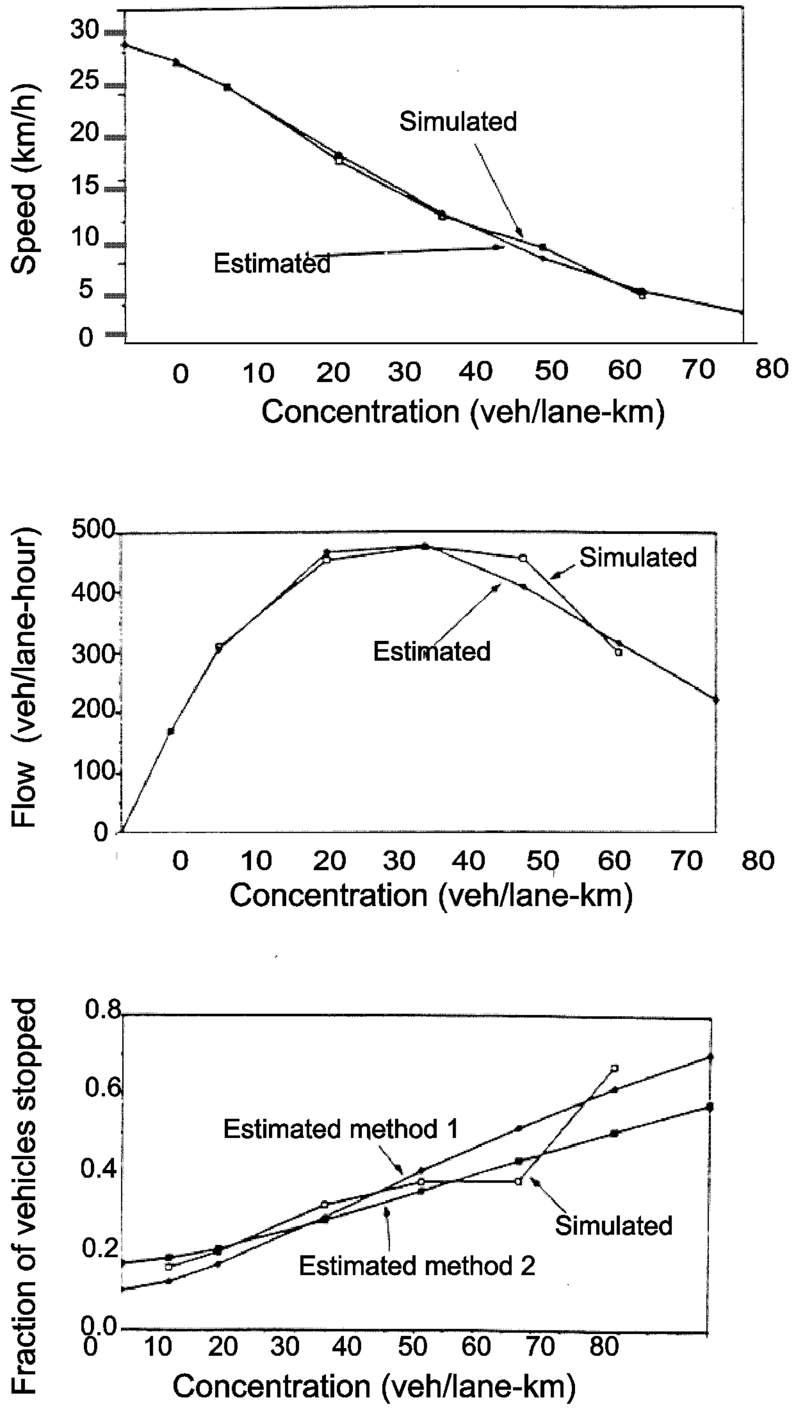


Figure 6.26
Comparison of Model System 3 with Observed Simulation Results
(Williams et al. 1987, Figures 12-14).

6.5 Concluding Remarks

As the scope of traffic control possibilities widens with the development of ITS (Intelligent Transportation Systems) applications, the need for a comprehensive, network-wide evaluation tool (as well as one that would assist in the optimization of the control system) becomes clear. While the models discussed in this chapter are not ready for easy implementation, they do have promise, as in the application of the two-fluid model in San Antonio (Denney et al. 1993; 1994).

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