Quality and Theory
of
Traffic Flow

A Symposium
by
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BUREAU OF HIGHWAY TRAFFIC
YALE UNIVERSITY
Foreword

Motor vehicles are creating demands for an unprecedented amount of urban street construction and new traffic controls in addition to work on the Interstate system of highways. It is necessary for the prudent administrator to set up systems by which the relative merits of projects may be evaluated and priority of construction planned since it is impossible that all of the needed projects be undertaken simultaneously.

Any such evaluation system will consider items such as structural condition and safety together with a measure of service to the road user. It is with the last of these, service to the road user, that the first two papers in this symposium are particularly concerned. What is an effective measure of the quality of traffic flow? Traffic volume, speed and nature of traffic accidents have been studied extensively, but attempts to measure quality of flow and proposals of how this should be done are still in the development stage.

Traffic flow is a complex phenomenon. The last three papers of this symposium deal with the relationships of speed, travel times, volume and density. These relationships are important aspects of both the quality and theory of traffic flow. A better understanding of them may lead to improved methods of studying and analyzing traffic flows so cause may be related to effect for the control of road use and for further development of road capacity concepts.

The obvious purpose of this symposium is to interest others in continued research in the quality and theory of traffic flow. Many small fragments of information as contained in these theses may lead to a more fundamental understanding of traffic operations.

Fred W. Hurd, Director
Bureau of Highway Traffic
Preface

That no single approach to the measurement of traffic service has been established is evident by the two methods proposed by the first two papers in this symposium. The first of these, by Greenshields, measures the "quality of traffic flow," while the second, by George, is a measure of "traffic congestion."

The two methods differ in their approach to field measurements. Greenshields uses a modification of the floating car technique in which a vehicle "floats" through the traffic and attempts to drive at a rate representing that of the average traffic stream over a length of highway. Qualities of the entire traffic stream are based on measurements from one or at the most several vehicles. George bases his measurements on observations of all the vehicles in the traffic stream, the limitation being that the observations are usually made at only one location and do not apply to a length of highway. It would be necessary to make observations at a series of locations in order to determine the congestion characteristics of a route.

Greenshields assumes that there are three factors that determine the quality of traffic flow which is expressed by a dimensionless "index" number. Average over-all speed is directly proportional to quality of flow and is therefore placed in the numerator of the expression representing quality of flow. In other words, the higher the over-all speed, the greater the quality of flow. On the other hand, annoyance and frustration are caused largely by forced changes in speed and the frequency of such changes. It is these latter two quantities which would appear to be inversely proportional to the quality of flow, and Greenshields places them in the denominator in his formula. According to this concept, quality of flow is basically equal to speed divided by speed change.

George assumes that traffic inefficiency is made up of two major factors: (1) time of operation or time loss, and (2) driver inconvenience and discomfort. The first factor, time loss, can be shown to relate to the density (vehicles per mile) over a period of time, which in turn is numerically equal to the total time of operation over a one-mile segment of roadway for all vehicles passing in one hour. Both Greenshields and George consider
travel time or travel speed as effective factors in the development of their ratings. Greenshields uses speed, George uses density, pointing out that it is related to total travel time.

The major portion of the Greenshields study was completed during the fall term 1954-55 while he was on leave from George Washington University and a member of the Bureau of Highway Traffic staff. This work commanded a considerable amount of interest among students at the Bureau during following years. This symposium includes not only the original Greenshields study but the student theses inspired by it during the academic years ending in 1956, 1958, 1959 and 1960.

The last three papers by Guerin, Palmer and Underwood are concerned with the theory of traffic flow but are closely related to the problems and discussions raised in the first two papers. They provide a closer look at the relationships among speed, travel time, volume and density.

It should be emphasized that each of the theses was individually developed and there has been no attempt to alter the original concepts of the authors by coordinating them. Thus, assumptions and definitions may differ among the various authors and the conclusions reached may not necessarily agree. However, in the interests of brevity, the theses have been severely edited with respect to repetitions and background information.
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The Quality of Traffic Flow

by

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BUREAU OF HIGHWAY TRAFFIC
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Chapter I-1
DEVELOPMENT OF THEORY AND METHODS
OF FIELD STUDY

This report presents the development of a measure for determining the level of quality of traffic movements, principally in urban areas. This measure is called the "quality" index of traffic flow. Chapter I is devoted to the development of the theory and a brief description of the method of collecting the field data. Chapter II presents the derivation of field data. Chapter III contains tabulations of the quality values obtained for a number of facilities and a discussion of their significance. Several applications are covered in Chapter IV. Conclusions are drawn in Chapter V.

Quality of Traffic Flow

The pertinent factors of quality of traffic flow are readily apparent. The most obvious and generally recognized factor is the average or overall speed. Average speed determines the time of travel. The higher the average speed within the limits of safety, the higher is the quality of travel.

The amount of change of speed in both acceleration and deceleration and the frequency of these changes that are necessary to meet fluctuating traffic conditions are undesirable factors of traffic flow. They are irritating to the driver and they increase the cost of operation. They might be called "frustration" factors.

It is known from experience that the three factors of average speed, change of speed and frequency of change are neither entirely dependent upon, nor independent of each other. Since they are not entirely dependent, none can be omitted.

It is also clear that quality of flow cannot be measured at a point, but must be measured over a sufficient distance for fluctuations in flow to occur. All measurements should be prorated to a unit distance. The most useful distance to select is one

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1 A preliminary report on the theory of this study, "Quality of Traffic Transmission," Bruce D. Greenshields, was presented at the Thirty-Fourth Annual Meeting of the Highway Research Board in Washington, D. C., in January 1955. The paper given at that time was published in the Highway Research Board Proceedings, Vol. 34, p. 508. The material herein briefly reviews the previously published theory and presents additional information on its application.
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mile because speeds are given in miles per hour and highway transportation is usually thought of in terms of vehicle miles.

The higher the average speed the more desirable the flow and, therefore, the higher the expected index number. The greater the amount of speed changes and their frequency the less desirable is the flow and, therefore, the lower the expected index number. Essentially the number is a function of speed divided by change of speed.

Symbolically, a proposed expression for quality might be written:

\[ Q = \int \frac{S}{\Delta s} \sqrt{f} \]

wherein \( \int \) stands for function, and where

\[ Q = \text{quality index} \]
\[ S = \text{average speed (miles per hour)} \]
\[ \Delta s = \text{absolute sum of speed changes per mile} \]
\[ f = \text{number of speed changes per mile} \]

It may be shown that the \( Q \) number for measuring the quality of flow is dimensionless. Flow does not occur at a point but over a distance; therefore, the index \( Q \) must be for some unit of length such as one mile. Since \( f \) is a pure number, the terms having dimensions are the average speed, \( S \), and change of speed per mile, \( \Delta s \). Letting \( T = \text{time} \) and \( L = \text{distance} \), the dimension equation for \( Q \) per unit of distance may be written as:

\[ \frac{Q}{L} = \frac{L_1}{T_1} \sum \left( \frac{L_2}{T_2} - \frac{L_3}{T_3} \right) \]

wherein \( \sum \) stands for summation. The subscripts indicate that the \( L \)'s and \( T \)'s have different numerical values. Dimensionally, \( \left( \frac{L}{T} - \frac{L}{T} \right) = \frac{L}{T} \); also, \( \left( \frac{L}{T} - \frac{L}{T} \right) \) never equals zero except when there would be no speed change and \( Q \) would be infinity. Therefore, the above becomes simply \( Q = \frac{L/T}{L/T} \) and \( Q \) is dimensionless for it is simply equal to speed divided by speed.

\(^2\) Only speed changes of 1 m.p.h. or more were recorded for this study. On expressways, smaller speed changes may have to be recorded.
THE QUALITY OF TRAFFIC FLOW

The frequency of changes in speed was found by field runs to be highly variable. It seemed desirable, therefore, to give less weight to the frequency than to average speed or speed change.

Furthermore, it is known that in accelerating and decelerating, more distance is required for a speed change in the order of two to 15 miles per hour than for a change from two to five miles per hour. This means that the number of changes per mile must decrease as the range of speed change increases.

Since small speed changes are not as annoying as large changes, and since the range decreases as the frequency increases, additional support was found for reducing the weight of the larger frequencies. This weighting was accomplished by taking the square root of the frequency; for example, the weight given to a frequency of 10 would be \( \sqrt{10} = 3.16 \). Some other weighting system, such as using the logarithm of the number, could have been adopted.

The proposed \( Q \) value thus becomes

\[
Q = \frac{S}{\Delta s \sqrt{f}}
\]

It was found that the \( Q \) values using this ratio were very small. For example, a typical value might be

\[
Q = \frac{S}{\Delta s \sqrt{f}} = \frac{8.2}{355 \sqrt{28}} = 0.00437
\]

To avoid small numbers the constant \( K = 1000 \) was introduced into the formula. Thus, the formula becomes:

\[
Q = \frac{KS}{\Delta s \sqrt{f}}
\]

In the above example, \( Q \) would equal

\[
\frac{1000 \times 8.2}{355 \sqrt{28}} = 4.37
\]

The chief characteristic of the quality index number is that it is basically equal to speed divided by change of speed (or velocity) and is dimensionless. Within this framework many variations are possible. For example, it has been suggested that drivers are more annoyed by the amount and number of speed changes per unit of time than per unit of distance. Making this
QUALITY AND THEORY OF TRAFFIC FLOW

change does not alter the nature of the index number.\(^1\)

There are other changes that come to mind. Delay or stopped
time might be thought of as another variable, but in the develop-
ment of the index a stopped vehicle is considered to be one trav-
eling at zero speed. Inserting the number of stops per mile
would not change the nature of the quality index.

The formula developed here pertains specifically to urban
operators. However, the hypothesis that quality of traffic flow
can be expressed as a function of speed divided by change in speed
is held to be generally applicable on all types of highways.

Methods of Collecting Field Data

Ideally, the field data should include a count of all vehicles
together with a continuous record of the speed of each vehicle to
obtain a Q value for a given facility. Since such a complete rec-
ord is almost impossible to obtain, it was logical to try to secure
a random sample sufficiently large to be reliable. Two methods
seemed feasible. One was to use a "test" car driven with the
traffic and equipped with a recording speedometer; the other was
to use time-lapse pictures.

In the "test car" method, it was found that using both the
"average" car and the "floating" car gave reliable overall travel
times and that the degree of accuracy depended on the size of the
sample. (The "floating" car driver paces average speed of the
stream by passing as many vehicles as pass him.) It was demon-
strated that either the "average" car or the "floating" car speeds
represent the overall running speed of the traffic stream quite
accurately and with practically the same required sample size.
If the overall speeds are correct, then it is logical to assume
that the continuous speed record, as given by the test car, is
representative of the traffic stream. The summation of the vary-
ing speeds throughout a given distance must equal the overall
speed. To facilitate the collection of data by the "test car"
method, a recording speedometer\(^4\) was utilized to record instan-
taneous speeds automatically and to code other data when push-
buttons were actuated. The instrument is shown in Figures I-1
and I-2.

\(^1\) If the change suggested is made, the dimension equation becomes:

\[ Q = \frac{L/T}{L/T \cdot L/T} = \frac{L/T}{L/T} + \frac{L/T}{L/T} = \text{dimensionless number} \]

\(^4\) The recording speedometer was conceived at the Bureau of Highway Traffic and
was developed by the Automatic Signal Division of Eastern Industries, Inc.,
Norwalk, Connecticut.
FIGURE 1.1
RECORDING SPEEDOMETER
THE QUALITY OF TRAFFIC FLOW

FIGURE 1.2
RECORDING SPEEDOMETER INSTALLED IN CAR

stopped vehicle to secure methods with the travel of the

standard-vehicle

1 quite small size, is

FIGURE 7
urban mile

vehicle to

re-
QUALITY AND THEORY OF TRAFFIC FLOW

The other method of collecting data consisted of taking aerial time-lapse pictures to secure what might be called a moving average sample. The pictures used in this study were taken from a light plane flying at about sixty miles per hour and at an altitude of approximately 1,000 feet. An area about 800 feet square was covered by the pictures taken at the 1,000-foot altitude. A frame counter and watch were included in each picture.

A 35-mm movie camera, equipped with a wide-angle lens and a modified shutter to give single-frame exposures of 1/200 second was used.

As a check on the aerial photographic method, as well as a means of recording traffic conditions, a series of ground runs were made with a passenger car equipped with a special camera normally used to record the performance of commercial drivers. This type of camera is bolted to the ceiling of the car on the right side and takes pictures through the windshield. The camera takes both moving and still pictures, and is operated by the actuation of a button on the steering wheel. A chronometer and a speedometer reading are included in each picture.

Figure I-3 shows a sequence of aerial pictures of King Street in Alexandria, Virginia. Figure I-4 shows pictures taken at the same time with the special camera installed in an automobile. In each pair, the ground view corresponds with the aerial view. The ground pictures were taken at a rate of about twelve per minute.

While the speeds obtained from the speedometer in the ground pictures agreed within two miles per hour with the speeds obtained from the aerial pictures, the analysis of data obtained by the aerial-picture method was much more tedious than the recording-speedometer method and it is not recommended for urban traffic. For rural traffic, however, or in outlying urban districts, traffic flows too freely to obtain data by recording speedometer since there is no average or typical vehicle. Under these conditions, the aerial-picture method seems to be the most practical since in free flowing traffic it is necessary to observe all, or nearly all, vehicles to obtain an accurate flow pattern.

5 The Markel Service, Inc., camera.
THE QUALITY OF TRAFFIC FLOW

FIGURE I-3 — AERIAL PICTURE

FIGURE I-4 — GROUND PICTURE

KING STREET, ALEXANDRIA, VIRGINIA
Chapter I - II
DEVELOPMENT OF FIELD DATA

After the field data were recorded, they were analyzed by statistical methods. A difficulty arose that had to do with the form of the index number, \( Q = \frac{KS}{\Delta_s \sqrt{f}} \), which has the nature of a rate. Rates cannot be averaged arithmetically. The method of analysis used to overcome this problem will be shown by the derivation of a \( Q \)-value for a typical street.

The statistical method of reporting results is recommended since it shows the accuracy of the results obtained. The standard procedures for finding these values may be found in any book on statistical methods. However, they are included here for convenience.

Nonadditive Property of \( q \)-Values

That the average value of two or more \( q \)-values should not be an arithmetic average is easily demonstrated as follows:

Let \( q \) equal the quality value for one run, \( Q \) the arithmetic average of a number of runs, and \( Q \) the value obtained by considering a number of short runs as one long run.

Let the factors for one mile be \( S = 10 \) m.p.h., \( \Delta_s = 200 \) m.p.h., and \( f = 25 \). For the succeeding mile, let \( S = 12 \) m.p.h., \( \Delta_s = 180 \) m.p.h., and \( f = 16 \). With these values, for the first mile,

\[
q_1 = \frac{1000 \times 10}{200 \sqrt{25}} = 10
\]

and for the second mile,

\[
q_2 = \frac{1000 \times 12}{180 \sqrt{16}} = 16.67
\]

the arithmetic average,

\[
\overline{Q} = \frac{q_1 + q_2}{2} = \frac{10 + 16.67}{2} = 13.34
\]

---

\(^1\) The quality number obtained from a single test car run or from the several cars observed on one plane run will hereafter be designated by "q."
THE QUALITY OF TRAFFIC FLOW

The $Q$ value is obtained by considering the two miles as one run. Note, again, that all the factors must be prorated to a one-mile base. Thus, for the two miles,

$$S = \frac{10 + 12}{2} = 11 \text{ m.p.h.}$$

$$\Delta_s = 200 + 180 = 380 \text{ m.p.h.}$$

$$f = 16 + 25 = 41$$

Prorated to one mile,

$$S = 11 \text{ m.p.h.}$$

$$\Delta_s = 190 \text{ m.p.h.}$$

$$f = 20.5$$

Therefore, for the two-mile run,

$$Q = \frac{1000 \times 11}{190 \sqrt{20.5}} = 12.78$$

The non-additive property of $q$-values presents an unusual problem, but fortunately one that can be avoided. As just shown, this may be accomplished by treating all the runs as one long run. This procedure also avoids the difficulty of getting a stable quality value from very short runs. Since the factors for all the runs are added before combining to get $Q$, a wide fluctuation in any one run does not unduly affect $Q$.

While this method is reliable for obtaining a $Q$-value, it gives no information as to the statistical stability of $Q$. Interest is retained in the $q$-values for individual runs as well as in the mean $Q$ of all runs, for the purpose of establishing statistical significance of the $Q$-value.

Development of $Q$-Values for a Typical Street

The method of statistical analysis may be demonstrated by working out the $Q$-values for a typical urban street. The development of values for a section of Whitney Avenue in New Haven, Connecticut, is presented as an example. The character of the street is shown by the photograph, Figure I-5.

Whitney Avenue is a major radial route. The section studied extends from Bradley Street at the outer edge of the city's central area to Cliff Street, a distance of 6,950 feet. It is predominantly residential in character. Many apartment buildings and a
few office buildings and business establishments are located in the section. The pavement is 60 feet wide except for one block, between Bradley and Sachem Streets, which has a width of 42 feet. Parking is permitted on both sides of the street except between Bradley and Sachem Streets. Traffic signals are located at four intersections from 1,600 to 2,000 feet apart. The signals are not interconnected and are operated on a 60-second cycle with a 50-50 split. During the hours studied, the traffic volumes in one direction ranged from 350 to 968 vehicles per hour.

A total of 95 runs was made over this section of Whitney Avenue selected for study. From the data obtained, quality of flow values were found as follows:

- Range of $q$-values = 22.7 to 288.0
- Arithmetic mean, $\bar{Q} = 80.0$
- Standard deviation, $\sigma = \pm 53.2$
- Standard error of the mean, $S = \pm 5.49$
- Quality index, $Q$ (true mean) $\bar{Q}/Q = 62.6$
- Deviation from true mean, $\sigma' = \pm 56.0$
- Error of the true mean, $S_{\bar{Q}} = \pm 5.78$
THE QUALITY OF TRAFFIC FLOW

For ease in calculating the mean and the standard deviation, the q-data for this example are grouped as shown in Table I-I.

The first column in the table gives the class limits of q-values whose class interval (C) is 20; the second column gives the number of q-values (f) in each class; the third column gives the deviation in class intervals (t) from a mid-value q_o; the fourth column is the product ft; the fifth column is ft^2, the product of the (t) and (ft) columns.

The arithmetic mean is equal to

$$\bar{Q} = q_o + C \frac{\sum ft}{N}$$

where N is the total number of q-values. Thus,

$$\bar{Q} = 150 + 20 \frac{(-332)}{95} = 80.0$$

The standard deviation is equal to

$$\sigma = C \sqrt{\frac{\sum ft^2}{N} - \left(\frac{\sum ft}{N}\right)^2}$$

Thus, in this example,

$$\sigma = 20 \sqrt{\frac{1832}{95} - \left(\frac{-332}{95}\right)^2} = 53.2$$

The magnitude of the standard deviation is such that approximately two-thirds of the values fall less than one standard deviation away from the arithmetic mean, and such that roughly 95 percent of the values lie less than two standard deviations away from the mean.

The standard error of the mean

$$S_{\bar{Q}} = \frac{\sigma}{\sqrt{N - 1}}$$

$$S_{\bar{Q}} = \frac{53.2}{\sqrt{95 - 1}} = \pm 5.49$$

As described previously, Q is obtained by taking the average for all runs of each of the three factors that go to make up quality of flow and substituting the values found in the Q formula. The resulting number is referred to as Q (true mean) or just as Q.
TABLE I-1
CALCULATION OF ARITHMETIC MEAN AND STANDARD DEVIATION
OF q-VALUES

<table>
<thead>
<tr>
<th>Class Intervals of q-Values</th>
<th>Number of q-Values</th>
<th>Deviation in Class Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q</td>
<td>f</td>
</tr>
<tr>
<td>20 - 39.9</td>
<td>13</td>
<td>-6</td>
</tr>
<tr>
<td>40 - 59.9</td>
<td>28</td>
<td>-5</td>
</tr>
<tr>
<td>60 - 79.9</td>
<td>26</td>
<td>-4</td>
</tr>
<tr>
<td>80 - 99.9</td>
<td>9</td>
<td>-3</td>
</tr>
<tr>
<td>100 - 119.9</td>
<td>6</td>
<td>-2</td>
</tr>
<tr>
<td>120 - 139.9</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>140 - 159.9</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>160 - 179.9</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>180 - 199.9</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>200 - 219.9</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>220 - 239.9</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>240 - 259.9</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>260 - 279.9</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>280 - 299.9</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

\[ C = 20 \quad N = 95 \quad ft = -332 \quad ft^2 = 1832 \]

\[ q_o = 150 \]
THE QUALITY OF TRAFFIC FLOW

In this example, \( Q = 62.6 \).

The difference between \( \overline{Q} \) and \( Q \) is

\[
d = \overline{Q} - Q = 80.0 - 62.6 = 17.4
\]

By use of a transfer formula the deviation from the true mean is found to equal

\[
\sigma' = O^2 + d^2
\]

\[
\sigma' = \sqrt{(53.2)^2 + (17.4)^2} = \pm 56.0
\]

The error of the true mean is expressed by

\[
S_Q = \frac{\sigma'}{\sqrt{N - 1}}
\]

\[
S_Q = \frac{56.0}{\sqrt{95 - 1}} = \pm 5.78
\]

There is approximately a two-thirds assurance that the true value of \( Q \) lies between 62.6 ± 5.78 and a 95 per cent assurance that it lies between 62.6 ± 2 x 5.78 or 62.6 ± 11.56.
Chapter I - III

TYPICAL VALUES OF QUALITY OF TRAFFIC FLOW

Quality of flow values were derived from speedometer records for Whitney Avenue, U. S. Route 1, and Chapel Street, all in New Haven, and for 35th Street and 36th Street, First Avenue and Second Avenue in New York City. Quality values for King and Duke Streets in Alexandria, Virginia, were derived from time-lapse photography. Values for the George Washington Bridge were derived from both recording speedometer data and time-lapse photographs.

The following pages give information on the general character of the streets studied, traffic regulations and controls in force, and amount of data collected. The values of quality of flow are reported together with measures of their statistical stability. A subjective rating of the quality of flow, whether poor, fair or good, is also given.1

The information is presented in order of increasing quality of flow as based on this subjective rating.

The photographs included do not necessarily depict traffic characteristics typical of the several streets, but will give an impression of their appearance.

On the George Washington Bridge, a check of q-values was made as obtained from time-lapse pictures against values obtained from speedometer records. A comparison of eight test-car runs made when traffic volumes were about the same as those recorded by aerial pictures gives results which are surprisingly close, as shown below:

<table>
<thead>
<tr>
<th>Test Car Method</th>
<th>Aerial Pictures Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Speed in m.p.h.</td>
<td>40.21</td>
</tr>
<tr>
<td>QUALITY INDEX (Q)</td>
<td>673.12</td>
</tr>
</tbody>
</table>

This would indicate that one method may be substituted for the other although, as pointed out in Chapter I, consideration should be given to problems of analysis in the selection of method.

1 Information on Whitney Avenue, New Haven, has already been presented in Chapter II and will not be repeated.
THE QUALITY OF TRAFFIC FLOW

Scale of Q-Values

The following pages show the wide range of quality of flow values found on the several streets studied and the high degree of variability of quality on most of them. The range of values from 1/2 to well over 1,000 indicates that the quality index is a very sensitive measure. It shows differences that cannot be detected by the driver. The sensitivity is very useful in comparing similar streets. However, the scale of the quality index value is not a direct reading or arithmetic scale but rather it follows a logarithmic or exponential relationship. It is for this reason that most of the graphic presentations of Q values in this report are plotted on a semi-logarithmic or ratio scale. The quality of flow index loses its significance and is overly sensitive at very high values. It is doubtful that there is any real difference between an index of 5,000 and one of 10,000. Theoretically, the upper limit for the quality index value is infinity (with no speed change). However, this is meaningless. Perhaps all values above, say, 4,000 or 5,000 should be regarded as equal.

The present study makes no attempt to develop standard values or ranges of values for the several types of streets. Such a study would require gathering of thousands of samples from possibly hundreds of streets under many different prevailing traffic regulations, abutting land use types and traffic volume and density conditions. With a large amount of data available it might be possible to develop a new scale of quality values not subject to the difficulties mentioned above. Perhaps the extreme refinements of the current scale are not justified and the units of the new scale could be considerably "blunted" in terms of the calculated Q values. A proposed scale with application to the following examples is shown in Table I-II.
QUALITY AND THEORY OF TRAFFIC FLOW

TABLE 1-II
DEGREE OF QUALITY SCALE
(With Applications)

<table>
<thead>
<tr>
<th>Degree of Quality</th>
<th>Range of Q's</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>over 3750</td>
</tr>
<tr>
<td>2</td>
<td>1500-3750</td>
</tr>
<tr>
<td>3</td>
<td>600-1500</td>
</tr>
<tr>
<td>4</td>
<td>250-600</td>
</tr>
<tr>
<td>5</td>
<td>100-250</td>
</tr>
<tr>
<td>6</td>
<td>40-100</td>
</tr>
<tr>
<td>7</td>
<td>16-40</td>
</tr>
<tr>
<td>8</td>
<td>6.5-16</td>
</tr>
<tr>
<td>9</td>
<td>2.5-6.5</td>
</tr>
<tr>
<td>10</td>
<td>under 2.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Street</th>
<th>Subjective Rating</th>
<th>Ave. Overall Speed (mph)</th>
<th>Volume During Study Period (veh/hr)</th>
<th>Q</th>
<th>Degree of Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>35th &amp; 36th Streets</td>
<td>Poor</td>
<td>4.8</td>
<td>204-350</td>
<td>1.6</td>
<td>10</td>
</tr>
<tr>
<td>King Street</td>
<td>Poor</td>
<td>7.4</td>
<td>400</td>
<td>1.5</td>
<td>10</td>
</tr>
<tr>
<td>Chapel Street</td>
<td>Poor</td>
<td>7.8</td>
<td>237-458</td>
<td>4.4</td>
<td>9</td>
</tr>
<tr>
<td>U. S. Route 1</td>
<td>Poor</td>
<td>13-14</td>
<td>565-964</td>
<td>7.0</td>
<td>8</td>
</tr>
<tr>
<td>Duke Street</td>
<td>Fair</td>
<td>20.3</td>
<td>500</td>
<td>11.7</td>
<td>8</td>
</tr>
<tr>
<td>First &amp; Second Avenues</td>
<td>Fair</td>
<td>15.2</td>
<td>Not available</td>
<td>16.3</td>
<td>7</td>
</tr>
<tr>
<td>Whitney Avenue</td>
<td>Fair</td>
<td>22.4</td>
<td>350-968</td>
<td>62.6</td>
<td>6</td>
</tr>
<tr>
<td>George Washington Bridge</td>
<td>Good</td>
<td>42.2</td>
<td>Not available</td>
<td>872.8</td>
<td>3</td>
</tr>
</tbody>
</table>
35th and 36th Streets, New York City

Subjective Rating: Poor

35th and 36th Streets traverse the New York garment district which is served by trucks and pushcarts and is severely congested during business hours.

Land Use: Commercial

Width: Approximately 30 feet

Parking: Permitted on both sides (thus allowing only a single lane for moving traffic)

Signals: At every intersection; fixed time

Other Controls: Both streets are one-way; 35th westbound, 36th eastbound

Number of Runs: 29 runs, both directions. Method: Recording Speedometer

Volume: From 204 to 350 per hour in one direction

Average Over-all Speed: 4.77 m.p.h.

Quality of Flow:

Range of quality index values for individual runs = 0.44 to 6.24
Arithmetic mean, $\bar{Q}$ = 1.96
Standard deviation, $\sigma$ = ± 1.55
Standard error of the mean, $S$ = ± 0.29
Quality Index, $Q$ (true mean), $\bar{Q}$ = 1.56
Deviation from true mean, $\sigma'$ = ± 1.60
Error of the true mean, $S_{\bar{Q}}$ = ± 0.30
QUALITY AND THEORY OF TRAFFIC FLOW

FIGURE 1-7
KING STREET, ALEXANDRIA, VIRGINIA

SUBJECTIVE RATING: POOR

From: Union Street  To: Harvard Street  Distance: 4,913 feet
King Street is a main shopping street in Alexandria, Virginia.
Land Use: Predominantly commercial
Width: 37 feet
Parking: Permitted, both sides
Signals: At each intersection; intersections are uniformly spaced, 313 feet apart
Number of Runs: Three flights. Method: Aerial time-lapse photo
Volume: Approximately 400 per hour; both directions
Average Over-all Speed: 7.42 m.p.h.

QUALITY OF FLOW:
Range of quality index values for individual flights = 1.31 - 1.50
QUALITY INDEX (Q) = 1.485
(other measures not obtained)
THE QUALITY OF TRAFFIC FLOW

FIGURE 1-8

CHAPEL STREET, NEW HAVEN, CONNECTICUT

SUBJECTIVE RATING: POOR

From: Park Street  To: Olive Street  Distance: 4,270 feet
Chapel Street is the main east-west street through the central business district; the section studied traverses the main shopping area and the fringe areas lying on either side of it.

Land Use: Commercial
Width: 42.3 feet average
Signals: At nine (all) intersections; part of fixed time central business district system
Number of Runs: 60 runs, both directions.  Method: Recording Speedometer

Volume: From 237 to 458 per hour in one direction
Average Over-all Speed: 7.84 m.p.h.

QUALITY OF FLOW:

Range of quality index values for individual runs = 1.74 - 11.79
Arithmetic mean, \( \bar{Q} \) = 4.96
Standard deviation, \( \sigma \) = \( \pm 2.18 \)
Standard error of the mean, \( S \) = \( \pm 0.41 \)
QUALITY INDEX, \( Q \) (true mean) \( \bar{Q} \) = 4.43
Deviation from true mean, \( \sigma' \) = \( \pm 2.24 \)
Error of the true mean, \( S_Q \) = \( \pm 0.42 \)
QUALITY AND THEORY OF TRAFFIC FLOW

FIGURE I-9
U.S. ROUTE 1, NEW HAVEN, CONNECTICUT
SUBJECTIVE RATING: POOR

From: Howard Avenue  To: East Street  Distance: 7,180 feet

U.S. Route 1 is a major through-traffic street which by-
passes the central business district; the section studied con-
ists of portions of Columbus Ave., Union Ave., and Water
St.; a short distance of one-way operation is included.

Land Use: Chiefly commercial and industrial; some tenant-
ment houses at the westerly end

Width: 36-40; (including one-way sections)

Parking: Permitted, except for short prohibited areas

Signals: At 13 intersections; generally spaced 700-900 feet apart;
but also additional intermediate intersections; fixed time;
progressively timed on most of route

Number of Runs: 34 runs, both directions. Method: Recording
Speedometer

Volume: From 565 to 964 per hour in one direction

Average Over-all Speed: 13-14 m.p.h.

QUALITY OF FLOW:

Range of quality index values for
individual runs
Arithmetic mean, \( \bar{Q} \)
Standard deviation, \( \sigma \)
Standard error of the mean, \( S \) \( \bar{Q} \)
QUALITY INDEX, \( Q \) (true mean) \( \bar{Q} \)
Deviation from true mean, \( \sigma' \)
Error of the true mean, \( S_Q \)

= 3.8 - 24.9
= 11.02
= \( \pm 5.55 \)
= \( \pm 0.96 \)
= 7.04
= \( \pm 6.82 \)
= \( \pm 1.17 \)
THE QUALITY OF TRAFFIC FLOW

FIGURE I-10
DUKE STREET, ALEXANDRIA, VIRGINIA

SUBJECTIVE RATING: FAIR

From: Harvey Street  To: Callahan Drive  Distance: 3,897 feet
Duke Street is an east-west arterial street approaching downtown Alexandria from the west
Land Use: Residential and roadside commercial
Width: 40 feet
Parking: Permitted on both sides
Signals: One signal at intersection with Peyton and Commerce Streets
Number of Runs: Nine flights. Method: Aerial time-lapse photo
Volume: Approximately 500 per hour; both directions
Average Over-all Speed: 20.26 m.p.h.

QUALITY OF FLOW:
Range of quality index values for individual flights = 6.42 - 16.04
QLAILITY OF INDEX, Q = 11.66
(other measures not obtained)
QUALITY AND THEORY OF TRAFFIC FLOW

FIGURE I-11
FIRST AND SECOND AVENUES, NEW YORK CITY SUBJECTIVE RATING: FAIR

From: Fifty-ninth Street To: 110th Street Distance: 13,440 feet
First and Second Avenues are major north-south arterial streets in Manhattan

Land Use: Commercial and residential (upper floors)
Width: First Avenue - 68 feet; Second Avenue - 60 feet
Parking: Permitted on both sides of both streets
Signals: At nearly all intersections; flexible - progressive system
Other Controls: Both streets are one-way; First, northbound; Second, southbound
Average Over-all Speed: 15.2 m.p.h.
Number of Runs: 30 runs, both directions. Method: Recording Speedometer

QUALITY OF FLOW:
Range of quality index values for individual runs = 3.27 - 126.37
Arithmetic mean, $\bar{Q}$ = 26.16
Standard deviation, $\sigma$ = $\pm$ 25.85
Standard error of the mean, $S$ = $\pm$ 4.43
QUALITY INDEX, $\bar{Q}$ (true mean) = 16.26
Deviation from true mean, $\sigma$' = $\pm$ 27.16
Error of the true mean, $S_Q$ = $\pm$ 4.62
THE QUALITY OF TRAFFIC FLOW

FIGURE 1-12

GEORGE WASHINGTON BRIDGE, NEW YORK CITY • SUBJECTIVE RATING: GOOD

From: Tower To: Tower Distance: 3,500 feet

The George Washington Bridge spans the Hudson River between New York City and New Jersey. It is about 1.4 miles long. The portion studied was the main span, 3,500 feet in length, between the two main towers of the bridge.

Width: 85 feet; eight lanes, two of which are reversible so that as many as 5 lanes can be operated in one direction.

Traffic Controls: None, except for speed limit of 40 m.p.h.

Number of Runs: 34. Method: Recording Speedometer. Aerial time-lapse pictures were also taken from the towers of the bridge.

Average Over-all Speed: 42.2 m.p.h.

QUALITY OF FLOW:

Range of quality index values for individual runs

\[ = 214.02 - 6705.0 \]

QUALITY INDEX (Q)

\[ = 872.79 \]
Chapter I-IV
SOME APPLICATIONS OF THE QUALITY INDEX

The Efficiency Index

One of the most obvious uses of the quality index is in determining the efficiency of street and vehicle operation. The efficiency of a street in transmitting traffic is measured by the quantity and quality of traffic flow. For a given street area and a given number of vehicles, the more vehicle-miles produced and the higher their quality the greater is the overall efficiency.

The efficiency of traffic flow may be expressed as the product of vehicle-miles produced per unit area and the ratio of relative costs of operation based on Q-values (called the quality ratio). Letting $V_m = \text{vehicle-miles}$, and $Q_r = \text{the quality ratio}$, the efficiency index is:

$$E = V_m \times Q_r$$

In determining $V_m$, the logical unit area is a lane one mile long. With this unit area the volume per hour and the vehicle-miles per hour are numerically equal. It is also convenient to select ten feet as the standard width of lane for multiples of ten are easily calculated.

To find $Q_r$, it is necessary to develop the following discussion on costs.

The Efficiency Index in Terms of Cost

In expressing the quality ratio, it is necessary to assign cost values to the several factors of the quality index. These, as already stated, are (1) the speed, (2) the sum of the speed differentials per mile, and (3) the square root of the frequency of speed changes per mile. As previously indicated, the product of the latter two factors may be thought of as a measure of annoyance or frustration. It is also a measure of the turbulence or unevenness of flow. The values which are used in the following sections are only approximate. It is judged, however, that they are accurate enough to indicate relative efficiencies.
THE QUALITY OF TRAFFIC FLOW

Calculations for Table of Cost Values

In making the calculations for the cost values shown in Figure I-13, certain assumptions and estimates must be made. The curve will vary according to the assumptions made.

The Q-values shown in column 1 of Table I-III are simply selected values to give sufficient points for drawing the curve. The speeds given in column 2 are based on the observed relationships of Q to speed.

The costs of vehicle operation, as shown in column 3, are based on the estimate that the cost varies from 5 to 8 cents per mile, and that the variation within this range is in a straight line relationship with Q and inversely proportional to it. The range is assumed to extend from Q = 1 to Q = 500. The cost is assumed to remain constant at 5 cents per mile beyond that point. As speeds increase in rural areas, the cost of operation would not continue to decrease with the increase in Q values.

The cost of operation is for passenger cars only. For mixed traffic, the cost of vehicle operation would be increased according to the proportion of trucks. For the purpose of illustrating how the cost versus Q curve is drawn, the vehicle cost of operation corresponding to a Q-value of 50 is calculated. This would equal 8 - \( \frac{50}{500} \times 3.00 = 8 - 0.3 = 7.7 \) cents.

Column 4 is calculated on the assumption that vehicle time is worth 2.25 cents per minute. The annoyance factors given in column 5 are taken from the observed relationship of Q to the annoyance factor \( \Delta_s \sqrt{f} \).

The annoyance costs given in column 6 are based on the assumption that the annoyance cost varies from a maximum of 35 per cent of the value of time at a point where Q = 1.00. This is assumed to be about the poorest quality of flow that is likely to be experienced. From this point, the annoyance cost is assumed to decrease in proportion as the annoyance factor decreases. For example, the cost at the point where Q = 1 is assumed to be 35 per cent of the time cost of 38.7 cents shown in column 4. This equals 13.5 cents. At the point where the annoyance factor equals 2,000 the cost is calculated to be \( \frac{2000}{3800} \times 13.5 = 7.1 \) cents. At speeds above 36 miles per hour the annoyance is assumed to be zero.

The total costs per mile, shown in column 7, are obtained by adding the costs in columns 3, 4 and 6.
<table>
<thead>
<tr>
<th>Q-Values</th>
<th>Speed in Miles per Hour</th>
<th>Cost of Vehicle Operation</th>
<th>Cost of Time (cents)</th>
<th>Annoyance Factor</th>
<th>Annoyance Cost</th>
<th>Total Cost Cents per Mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.5</td>
<td>8.00</td>
<td>38.7</td>
<td>3800</td>
<td>13.5</td>
<td>60.2</td>
</tr>
<tr>
<td>2</td>
<td>5.0</td>
<td>7.99</td>
<td>27.0</td>
<td>2800</td>
<td>9.9</td>
<td>44.9</td>
</tr>
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<td>4</td>
<td>5.0</td>
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<td>19.3</td>
<td>2000</td>
<td>7.1</td>
<td>34.4</td>
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<td>8.5</td>
<td>7.96</td>
<td>15.9</td>
<td>1500</td>
<td>5.3</td>
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</tr>
<tr>
<td>8</td>
<td>9.7</td>
<td>7.95</td>
<td>13.9</td>
<td>1430</td>
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<td>27.0</td>
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<tr>
<td>10</td>
<td>10.7</td>
<td>7.94</td>
<td>12.6</td>
<td>1270</td>
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</tr>
<tr>
<td>20</td>
<td>14.0</td>
<td>7.88</td>
<td>9.6</td>
<td>850</td>
<td>3.0</td>
<td>20.5</td>
</tr>
<tr>
<td>30</td>
<td>16.7</td>
<td>7.76</td>
<td>8.1</td>
<td>680</td>
<td>2.4</td>
<td>18.3</td>
</tr>
<tr>
<td>50</td>
<td>19.9</td>
<td>7.70</td>
<td>6.8</td>
<td>490</td>
<td>1.7</td>
<td>16.2</td>
</tr>
<tr>
<td>70</td>
<td>21.9</td>
<td>7.58</td>
<td>6.2</td>
<td>380</td>
<td>1.3</td>
<td>15.1</td>
</tr>
<tr>
<td>100</td>
<td>24.4</td>
<td>7.40</td>
<td>5.5</td>
<td>290</td>
<td>1.0</td>
<td>13.9</td>
</tr>
<tr>
<td>200</td>
<td>29.8</td>
<td>6.80</td>
<td>4.5</td>
<td>160</td>
<td>.7</td>
<td>12.0</td>
</tr>
<tr>
<td>400</td>
<td>33.3</td>
<td>5.60</td>
<td>4.1</td>
<td>60</td>
<td>.1</td>
<td>9.8</td>
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<tr>
<td>500</td>
<td>38.0</td>
<td>5.00</td>
<td>3.6</td>
<td>50</td>
<td>.0</td>
<td>8.6</td>
</tr>
<tr>
<td>1000</td>
<td>45.1</td>
<td>5.00</td>
<td>3.0</td>
<td>50</td>
<td>.0</td>
<td>8.0</td>
</tr>
</tbody>
</table>
Value of Time = $1.35 per Hour
Operation Cost = 5-8 Cents per Mile
Annoyance Cost = 35% of Time Cost

FIGURE 1-13
COMPARISON COST OF DRIVING WITH QUALITY OF TRAFFIC FLOW
QUALITY AND THEORY OF TRAFFIC FLOW

It is clear that similar analyses for any given composition of traffic could be developed. It is judged that annoyance costs should not be included for trucks. The major cost of truck operation is that of time. Truck drivers are expected to take the route by which they save time, regardless of the inconveniences of poor quality of traffic flow.

These approximate per-mile costs, corresponding to any Q-value, make it possible, once the Q-values have been obtained, to compare the cost of traffic operation on various routes. They also make it possible -- once acceptable standard Q-values have been established -- to obtain the quality ratio in terms of cost.

To show how to arrive at the quality ratio, an assumed example will be used. Suppose that the average volume has been observed to be 300 vehicles per hour per lane and that the observed Q-value is 50. This is below the quality of $Q = 125$ which has been assumed as a "standard" for a street having similar traffic and physical characteristics.

The quality ratio $Q_r$ may be obtained by use of Figure 1-13. For a $Q$ of 50 the cost per vehicle-mile is approximately 16 cents, while for 125, it is about 13 cents. Thus the $Q_r$ ratio $\frac{13}{16} = .81$.

In this case, the efficiency equals,

$$E = V_m \times Q_r$$
$$= 300 \times .81 = 243$$

The Quality Index and Gasoline Economy

The assumption that the cost of vehicle operation varied directly with the value of $Q$ was open to question. While it was clearly impossible within the time available to obtain data on the total cost of operation, which would include items such as repairs, tire wear and depreciation, it was possible to compare gasoline consumption with the quality of flow. This work was carried out as a thesis project by Roy E. Oakes during the spring of 1955, subsequent to the field work and analysis of the principal investigation.¹

It should be mentioned that there are other reasons for wanting to know the correlation of fuel consumption with the quality of flow. It is essential, in determining the engine efficiency of a motor vehicle, to know the complete speed performance characteristics under which the efficiency tests were run.

THE QUALITY OF TRAFFIC FLOW

The data on gasoline consumption were collected by making a series of test runs over Whitney Avenue and U. S. I. These routes have already been described in Chapters II and III.

Gasoline consumption was measured by a gasoline economy meter which was physically connected to one of the code pens of the recording speedometer. The gasoline consumption rates were obtained by dividing the lengths of the runs by the amount of gasoline consumed during the run. The results were thus expressed in miles per gallon. The distances were read from the recording speedometer chart by tallying the number of 100-foot blips. The amount of gasoline consumed was obtained by tallying the units of gasoline passed through the fuel meter, as recorded by the "fuel" pen. Calibration tests, made on the meter before and after the runs, showed that the average number of gasoline units per gallon was 998.

The gasoline economy may be expressed as a formula:

\[
\text{Gasoline economy} = \frac{d}{\frac{5280}{m}} = 0.189 \frac{d}{m}
\]

where \(d\) = distance in feet, and \(m\) = number of units per run.

Relationship Between Gasoline Economy and Q

A \(Q\)-value and the corresponding "gasoline economy" was computed for each of the 120 runs made with a 1949 car with standard transmission, and for the 120 runs made with a 1954 car having an automatic transmission. Several different drivers were used to eliminate the effect of the drivers, if any, on either the \(Q\)-value or the gasoline consumption. In interpreting the relationship, it was found that the effect of the driver was very minor and could be omitted.

To provide larger samples, the runs were broken up into short sections. This gave a wider range of \(Q\)-values, for the value of \(Q\) may vary widely from point to point along a street or highway. With the short runs the dispersion is quite wide, but the relationship between \(Q\) and gasoline economy is definitely established. The results of this analysis are shown by Figures 1-14 and 1-15 whose curves indicate that at higher speed and higher \(Q\)-values, the gasoline economy levels off or starts to decrease.

Since a plot on semi-log paper with its foreshortened scale does not give a true visual picture, the curves of Figures 1-14 and 1-15 were replotted on rectangular coordinate paper, Figure 1-16.
QUALITY AND THEORY OF TRAFFIC FLOW

It was necessary only to carry the Q-axis out to a value of 400, since the change in gasoline economy beyond this range was only 0.5 miles per gallon. The lower range of this curve shows the substantial increases in gasoline economy that correspond to a small increase in Q.²

The quality of flow values shown in Figures I-14, I-15 and I-16 were obtained from runs about 1-1/3 miles in length. On the basis that longer runs might give more stable values, the values in Figure I-17 were obtained by adding the short runs to obtain "runs" of about 20 miles. These Q-values ranging from about 10 to 100 show a maximum variation of about one mile per gallon for any particular Q-value. This is a much closer correlation than for the short runs. This indicates that for comparing fuel economy no run of less than about 25 miles should be considered.

The direct correlation between speed and the quality index and between fuel consumption and the index leads to the question of why speed or fuel consumption cannot be used in place of the quality index. However, since it is possible that the correlation of these factors with Q may vary widely under different operating conditions, the basic formula, \( Q = \frac{S}{\Delta S \sqrt{f}} \), must be employed if the concept of quality measured in terms of speed divided by change in speed is to prevail. Further testing will be needed to demonstrate stability (or lack of it) in the relationship between speed or fuel consumption and the quality index.

**Quality of Flow, Density and Volume**

The relationship between density and volume and the quality of traffic flow is essentially that of cause and effect. Volume and density are terms associated with "congestion" which is accepted to be a major cause of poor quality of flow. But while congestion is indefinite in meaning, volume is defined as the number of

² To determine whether a modified version of the quality index might have had a closer correlation with gasoline economy, the index was altered in two ways and tested. First, the frequency of speed changes per mile, \( f \), was omitted from the equation and the values for \( Q = KS/\Delta S \) were correlated with fuel economy. Secondly, index values equal to the average speed divided by the average range, \( R \), in speed change \( Q = \frac{KS}{R} \) were tried. The correlations obtained with these indices were no better than that given by the index \( Q = \frac{KS}{\Delta S \sqrt{f}} \). This index which is more sensitive to traffic variations and gives a more precise measurement of quality was, therefore, used in this application.
THE QUALITY OF TRAFFIC FLOW

of 400, was only 10% worse than the
5 and 1-
On the other hand, these values
obtain an about 10% lower
than the other economy
index.
A question of the relation
operating
loyed if
vided by
ceded to
between
quality
ime and
cepted
estion
ber of
we had a
two ways
mitted
el econ-
average
dined with
f. This
is precise
QUALITY AND THEORY OF TRAFFIC FLOW

LEGEND
- Driver A
- Driver B

QUALITY OF TRAFFIC FLOW - Q

GASOLINE ECONOMY - MILES PER GALLON

FIGURE 1-15
COMPARISON OF QUALITY OF TRAFFIC FLOW WITH FUEL ECONOMY
1954 CAR WITH AUTOMATIC TRANSMISSION

34
FIGURE I-15
COMPARISON OF QUALITY OF TRAFFIC FLOW WITH FUEL ECONOMY
1954 CAR WITH AUTOMATIC TRANSMISSION

LEGEND
- - 1949 CAR WITH STANDARD TRANSMISSION
- - 1954 CAR WITH AUTOMATIC TRANSMISSION

FIGURE I-16
COMPARISON OF QUALITY OF TRAFFIC FLOW WITH FUEL ECONOMY
FIGURE I-17
COMPARISON OF OVERALL QUALITY OF TRAFFIC FLOW WITH AVERAGE FUEL ECONOMY

EACH POINT REPRESENTS RUN OF APPROXIMATELY 20 MILES
THE QUALITY OF TRAFFIC FLOW

vehicles passing a point in one hour and density is defined as the number of vehicles per mile of roadway. Hence, for meaningful results, it is essential that we consider volume and density rather than congestion.

Obviously, since the closer vehicles are together and the faster they travel, the greater is the number that will pass a given point in a given time, we may write:

\[ V(\text{olume}) = D(\text{ensity}) \times S(\text{peed}) \]

or \[ D(\text{ensity}) = \frac{V(\text{olume})}{S(\text{peed})} \]

But the first expression indicates that identical volumes could prevail on any one street with different combinations of density and speed within limits. A low D and a high S could probably be associated with good quality while a high D together with a low S would tend to produce a poor quality. Thus, it would appear that within the limits in which \( D \) and \( S \) are independent of each other, volume may not be sensitive to changes in quality of flow. Conversely, quality of flow is independent of changes in volume to the extent that density and speed are independent of each other.

The relation of quality of flow with both volume and density on the several streets was investigated. Figure I-18 shows a plot of quality versus density for individual runs on portions of 35th and 36th Streets in New York. These street sections experienced moderate to extremely high densities and showed a reasonably good correlation with quality of flow. On the other hand, as discussed theoretically in the preceding paragraph, the plotted points for the several streets studied showed no discernible correlation between quality of flow and volume.
Chapter I-V
CONCLUSIONS

One of the major objectives of this study was to substitute measurement for opinion in characterizing the quality of traffic flow. It was necessary to devise a measure which would be sensitive to small variations in the quality of traffic flow. One of the requirements was that such a measure be a "pure" or dimensionless number similar to the several index numbers used in fluid mechanics which have done much to advance that science. The development of the traffic quality index, $Q$, satisfactorily meets these requirements.

The quality index, $Q$, is based on the concept that average speed is a positive factor in expressing the degree of traffic quality, while any variation in speed can generally be considered as turbulence, thus representing a negative factor in traffic flow quality. This resulted in the basic expression that $Q$ is equal to speed divided by change in speed, over a fixed distance. The present study has tested and found valid one particular form

$$Q = \frac{KS}{\Delta s \sqrt{f}}$$

of the basic expression in the case of typical urban streets. However, variations of the formula are expected to apply equally well to other types of traffic streams.

Two applications of the quality index, $Q$, were included in this report. One shows the use of the index in the development of relative cost factors of vehicle operation as a method of expressing the efficiency of an urban street. Another application of the quality index covered in this report was a test of the correlation between gasoline consumption and traffic flow quality.

One of the needs prerequisite to the wider use of the quality index is the development of acceptable standards applicable to streets of various types. While something of the range of the $Q$ numbers was learned in the study, it was also pointed out that they probably follow a ratio or logarithmic scale, or one in which equal increments along the scale have unequal significance. A vast amount of data will have to be gathered on many types of streets before a set of standard values or ranges can be set up. It was also demonstrated in this study that quality of traffic flow has a better correlation with traffic density than with traffic volume.
QUALITY AND THEORY OF TRAFFIC FLOW

Accuracy of measurement is prerequisite to advancement in any science. It is hoped that these first steps will lead to the development of more exact index measurements of the quality of traffic flow and that such measurements may receive wider practical use. It is through the use of precise measurements that standards of performance and optimum design are established.
Measurement and Evaluation of Traffic Congestion

by

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A Thesis Submitted in Partial Fulfillment of Requirements for a Certificate in Highway Traffic

Bureau of Highway Traffic
Yale University
May 1956
Chapter II - I
THE PROBLEM

Outline of the Problem

The problem of objectively measuring the way in which vehicular traffic flows on a section of highway requires some form of logical quantitative definition to provide the measuring tool.

For purposes of this paper it is assumed that traffic inefficiency is made up of two major factors: 1, 2

1. Time of operation or time loss.
2. Driver inconvenience and discomfort.

The concept of time losses naturally involves a difference in travel times and this in turn requires that there must be some basic or optimum operating condition for the purpose of determining time losses. It will be assumed therefore that optimum operating conditions exist, so far as uninterrupted flow is concerned, when traffic is operating with volumes permitting free moving conditions.

It is necessary to qualify volume as a measure of free moving conditions for it is obvious that either the density or the average speed must be specified in addition to volume. For example, 200 vehicles per hour would constitute congested conditions if the average speed of vehicles was, say, less than 5 m.p.h. With an average travel speed as low as 5 m.p.h., the average density of traffic with a volume of 200 vehicles per hour would be 40 vehicles per mile.

There is no inherent difficulty in determining the time of operation for the average vehicle over a given section of highway and the average density can be determined from the average volume of traffic and the average speed or, alternatively, the average travel time.

The establishment of a factor which measures the discomfort and inconvenience element of traffic operation presents a more complex problem than that for establishing a time-loss factor. The value which road users place on the opportunity to drive on

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freeways and expressways with uninterrupted travel transcends that of efficient vehicle operation. Conditions which permit travel with a minimum of interference from other vehicles are desired by the majority of drivers. This is exemplified by the fact that some drivers choose to travel on routes where the quality of flow is high even though the routes traversed involve greater travel times caused by longer distances. It is evident that comfort and convenience are important elements in the quality of flow and road users are quick to recognize the presence of these elements or the lack of them.

It has been found desirable to express the discomfort factor in those same units as the time-of-operation factor so that the terms of the congestion rating will be compatible with one another.

It is assumed in this study that the measure "Rating," as it will be termed henceforth, is given by:

$$R = \text{Density} + f\left(\frac{\text{Density}}{\overline{v}^b}\right)$$

where --

$$R = \text{Rating of traffic congestion}$$
$$f = \text{function of}$$
$$\overline{v} = \text{coefficient of variation of the distribution of speeds}$$
$$a \text{ and } b \text{ are constants}$$

The first term of this expression (Density), as will be shown, is related to the time of operation, and the second term,

$$f\left(\frac{\text{Density}}{\overline{v}^b}\right)$$

introduces the discomfort element. It will be noted that the whole expression is dimensionally acceptable and the terms are arithmetically additive.

With low densities and a consequential wide range in speeds, the congestion will be negligible and the expression gives a low rating. Conversely, with high densities and little freedom to move other than at the average speed, the congestion will be high and there will be a resulting high rating.

Time of Operation

It will be noted that by prorating measurements to the units of one mile and one hour, the magnitude of density is the same as
MEASUREMENT AND EVALUATION OF TRAFFIC CONGESTION

The number giving total time of operation for all vehicles. Density (the number of vehicles per mile) may be determined by:

Volume (vehicles per hour) × Average time to travel one mile (in hours)

The unit of density is vehicles per mile which is equal to --

\[
\frac{\text{Volume}}{\text{Average Speed}} \quad \text{where speed is the overall running speed.}
\]

In rural areas, and on urban expressways, where traffic is uninterrupted and variations in speed along a given section are due only to other vehicles on the road, it is probable that the average running speed can be deduced from an array of spot speeds with some acceptable degree of accuracy.

Attention is drawn therefore to the following definitions:

\[
\text{Time Mean Speed} = \sum \left( \frac{\text{distance}}{t} \right) \times \frac{1}{n}
\]

\[
\text{Space Mean Speed} = \left( \frac{\text{distance}}{\sum t} \right) \times n
\]

where \( t \) = travel time for each individual vehicle
\( n \) = number of observations

Space mean speed can be converted directly to travel time, but time mean speed cannot be readily converted. The arithmetic average of any array of spot speeds is the time mean speed, but an estimate of the space mean speed can be made. 4

Discomfort Factor

Other things being equal, high quality of flow is found when volumes and densities are low enough to permit the individual driver to travel in a manner which is not influenced to any appreciable extent by other vehicles on the highway. Under these optimum operating conditions there is a wide range in speeds and the coefficient of variation is of the order of 14 per cent for the majority of two-lane rural highways. 5

3 As an example: A volume of 1,200 vehicles per hour at a space mean speed of 60 m.p.h. represents a density of 26 vehicles per mile. At 60 m.p.h. the average travel time through a one-mile trap is one minute or, for all vehicles, 1,200 minutes or 20 hours, which is numerically equal to the density in veh./mile.


QUALITY AND THEORY OF TRAFFIC FLOW

For high-speed rural highways the coefficient of variation for free moving conditions is approximately 16 per cent. When the volumes are in the region of the possible capacity of a two-lane road, say at 1,800 v.p.h. and with congested conditions, the coefficient of variation is approximately 7 1/2 per cent.

In computing the values of the coefficient of variation it has been assumed that the distribution of speeds is very nearly normal and therefore a close approximation to the coefficient of variation is given by:

\[
\frac{100 \times (93 \text{ percentile} - 7 \text{ percentile})}{2.95 \times \text{average speed}}
\]

The relative magnitude of the discomfort factor can be measured by assuming that it is directly proportional to the density and indirectly proportional to the standard deviation of the distribution of speeds (or some fractional power of the standard deviation) or, alternatively, the coefficient of variation. In symbols a scale of factors would be given by:

\[
\frac{\text{density}}{d^b}
\]

or alternatively \[
\frac{\text{density}}{av^c}
\]

Where: \( d \) = standard deviation in miles per hour of the speed distribution of individual vehicles, \( v \) = coefficient of variation (per cent) of the speed distribution of individual vehicles, \( a, b \) and \( c \) are constants

**Alternative Methods of Measuring the Discomfort Factor**

In order to determine appropriate values of the constants \( a, b \) and \( c \) of the discomfort factor, alternative measures are discussed below. These measures are related to overtaking maneuvers and to lane-change opportunities.

It can be shown that to maintain desired speeds, the total number of overtaking maneuvers required to be performed is proportional to the square of the traffic volume, providing speeds follow a normal distribution.\(^6\)

MEASUREMENT AND EVALUATION OF TRAFFIC CONGESTION

The number of overtakings per mile per hour is then

\[
0.56 \frac{V^2 D_s}{S_s^2}
\]

Where:
- \( V \) = Volume of traffic in one direction (v.p.h.)
- \( S_s \) = Space mean speed of traffic (m.p.h.) with free speed distribution
- \( D_s \) = Standard deviation (m.p.h.) of traffic with free speed distribution

If it is assumed that the degree of inconvenience is related to the number of times that vehicles are required to undertake a passing maneuver in order to maintain desired speeds, the above relationship enables factors to be set up for varying traffic volumes. It is suggested that a particular factor for a given volume of \( V \), vehicles per hour in a single direction, would be given by computing the ratio:

\[
\frac{R_v - R_{vf}}{R_{vf}}
\]

Where:
- \( R_v \) = Required number of overtakings at volume \( V \)
- \( R_{vf} \) = Required number of overtakings at free speed volume

This line of reasoning has been pursued and the following scale of factors emerges:\footnote{Adapted from Committee on Highway Capacity, Highway Research Board. op. cit., p. 32.}

<table>
<thead>
<tr>
<th>v.p.h.</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>400</td>
<td>2</td>
</tr>
<tr>
<td>800</td>
<td>15</td>
</tr>
<tr>
<td>1,200</td>
<td>35</td>
</tr>
<tr>
<td>1,600</td>
<td>63</td>
</tr>
<tr>
<td>2,000</td>
<td>99</td>
</tr>
</tbody>
</table>

It could be argued that in the case of a two-lane highway the ratio of actual overtaking to desired overtaking maneuvers should be the criterion, but it is held that inconvenience is related to the number of times that drivers are obliged to take heed of other vehicles on the road, whereas the ratio of actual to required...
QUALITY AND THEORY OF TRAFFIC FLOW

overtakings is a measure of capacity.\(^1\)

Since the above argument does not involve the question of opposing flow it would appear to be applicable to either two-lane, two-way highways or multi-lane facilities.

Lane Change Opportunities

In the case of multi-lane facilities, another way of assessing the inconvenience factor is to assume that it is directly related to the opportunities that exist for making lane changes.

As volumes increase, the opportunity for lane change decreases at a rate which is governed by the maximum acceptable gap between vehicles in the lane into which the change is being made and the frequency of occurrence of gaps equal to or greater than the minimum.\(^8\)

On the assumption that vehicles are distributed in time and space randomly, it is seen that the distribution of such gaps is related exponentially to the volume. That is to say, the probability of gaps equal to or greater than \(t\) seconds is given by \(e^{-\lambda t}\) where \(\lambda\) is the volume in vehicles per second. However, the opportunity to accept gaps for a lane change exists only if the vehicle making the maneuver is in the proper position to accept.

If it is assumed that drivers on the average require one second ahead and behind in making a lane change, and the minimum acceptable gap is two seconds, it has been computed that lane changes are prevented from being performed for a two-lane, one-way traffic stream as follows:\(^9\)

<table>
<thead>
<tr>
<th>Total One-Way Volume</th>
<th>Per Cent of Time Lane Change is Prohibited</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>virtually nil</td>
</tr>
<tr>
<td>500</td>
<td>6</td>
</tr>
<tr>
<td>1,000</td>
<td>18.5</td>
</tr>
<tr>
<td>1,500</td>
<td>34</td>
</tr>
<tr>
<td>2,000</td>
<td>47</td>
</tr>
</tbody>
</table>

The percentages given above might therefore be taken as the relative values of the discomfort factor at various volumes.

\(^1\) O. K. Normann, "Results of Highway Capacity Studies," Public Roads, Vol. 23, No. 4, 1942, p. 70.


\(^9\) Ibid., p. 138.
MEASUREMENT AND EVALUATION OF TRAFFIC CONGESTION

Comparison of Methods of Measuring Discomfort

Each of the methods of measuring the relative magnitude of the discomfort factor has been plotted against corresponding volumes as shown in Figure II-1. It will be noted that the curve based on the ratio:

\[
density \sqrt{0.3 \times \text{coeff. of variation}}
\]

is in an approximate mean position to the other curves. Consequently it has been adopted in this study. It closely approximates the ratio:

\[
density \sqrt{\text{Standard deviation of speed distribution}}
\]

In order to keep the term with the dimension of density, the dimensionless coefficient of variation is preferred.

Evaluation of the Discomfort Factor

Little information is available concerning the value of comfortable travel or conversely the extent to which the average driver will directly or indirectly place measurable values on the discomfort factor.

Dr. Bruce D. Greenshields, in developing the measure of quality of traffic flow (presented in the first paper of this publication), assumed that the annoyance factor (that is, the discomfort factor) has a cost value that is at most 35 per cent of the value of the travel time. Dr. Greenshields makes the assumption that the cost value of annoyance (discomfort factor) varies uniformly from zero under optimum conditions, which are assumed to exist when the average speed is 35 m.p.h., to a maximum of 35 per cent of the cost value of travel time when the quality of flow is very low (average speed, 3 m.p.h. or thereabouts).

In this study, it will be assumed that the cost value of discomfort remains constant, and is 30 per cent of the magnitude of the cost of the time factor (in the Rating Equation) which, it will be remembered, has the appropriate dimensions of vehicle hours per mile.

Congestion Rating Formula

Having adopted the above relationship between travel time
QUALITY AND THEORY OF TRAFFIC FLOW

Curve 1  Based on Passing Needs.
Curve 2  Based on the ratio, \( \frac{\text{Density}}{\sqrt{0.3v}} \) where \( v \) is the coefficient of variation of speeds.
Curve 3  Based on the ratio, \( \frac{\text{Density}}{d} \) where \( d \) is the standard deviation of speeds.
Curve 4  Based on lane change opportunities.

FIGURE II-1
COMPARISON OF METHODS OF MEASURING DISCOMFORT FACTOR

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MEASUREMENT AND EVALUATION OF TRAFFIC CONGESTION

and the discomfort factor, the equation for the Rating of Congestion turns out to be:

\[
R = \text{Density} + \frac{0.3 \cdot \text{Density}}{\sqrt{0.3v}}
\]

or approximately

\[
R = \left(1 + \frac{1}{2 \sqrt{v}}\right) \times \text{Density}
\]

where \(v\) per cent = coefficient of variation of the distribution of travel speeds

and providing the value of the denominator of the discomfort factor is not less than unity.

The relationships between speed, volume and congestion rating are shown in Table II-I. These values are hypothetical and are based on the curves of the Highway Capacity Manual. Actual values, as measured in the field, are presented in the following chapters.

### TABLE II-I
HYPOTHETICAL DISCOMFORT FACTOR VS. VOLUME

<table>
<thead>
<tr>
<th>VOLUME Per Hour</th>
<th>MEAN SPEED Miles Per Hour</th>
<th>DENSITY Vehicles Per Mile</th>
<th>DISCOMFORT FACTOR</th>
<th>CONGESTION RATING</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>48.5</td>
<td>4.1</td>
<td>0.6</td>
<td>4.7</td>
</tr>
<tr>
<td>300</td>
<td>45.7</td>
<td>6.6</td>
<td>0.9</td>
<td>7.5</td>
</tr>
<tr>
<td>600</td>
<td>42.9</td>
<td>14.0</td>
<td>2.0</td>
<td>16.0</td>
</tr>
<tr>
<td>900</td>
<td>40.1</td>
<td>22.4</td>
<td>3.4</td>
<td>25.8</td>
</tr>
<tr>
<td>1,200</td>
<td>37.4</td>
<td>32.1</td>
<td>5.4</td>
<td>37.5</td>
</tr>
<tr>
<td>1,500</td>
<td>34.6</td>
<td>43.4</td>
<td>7.6</td>
<td>51.0</td>
</tr>
<tr>
<td>1,800</td>
<td>31.8</td>
<td>56.7</td>
<td>11.4</td>
<td>68.1</td>
</tr>
<tr>
<td>2,000</td>
<td>30.0</td>
<td>67.0</td>
<td>20.1</td>
<td>87.1</td>
</tr>
</tbody>
</table>
Chapter II - II
COLLECTION OF FIELD DATA

The preliminary discussion has indicated that ideally the information required in measuring the efficiency of traffic flow of a section of highway should comprise:

1. A continuous record of the travel speed of each vehicle.
2. A count of all vehicles.

Such a complete record is not practical to collect and, therefore, random sampling or, more correctly, stratified sampling of travel times and spot speeds has been adopted.

Sampling Procedure

Each set of observations was recorded over a five-minute period during which the time of travel, time gap (i.e., the time spacing) and the spot speed for every fourth vehicle were observed.

Equipment Used

(a) Travel Times. These were measured over a half-mile distance by the use of field telephones. An observer was stationed at each end of the section. As a vehicle passed the mark indicating the start of the section, its registration number was noted and a stop watch was started. As it traveled along the test section, its registration number was relayed by telephone to the observer at the other end of the section. The second observer relayed the passage of the vehicle past the second mark back to the first observer who stopped the stop watch.

The telephone technique was adopted in preference to the floating-car or test-car method mainly because it was possible to obtain a larger number of observations in a given period of time.

(b) Traffic Volumes. Volumes were obtained by automatic traffic counters supplemented by hand counters to manually record volumes in the direction of lighter flow. All five-minute-period volumes were multiplied by 12 to give corresponding hourly volumes.

(c) Spot Speeds. Spot speeds of vehicles were obtained by an instrument developed by the Road Research Laboratory of Great
MEASUREMENT AND EVALUATION OF TRAFFIC CONGESTION

Britain. Rubber detector tubes placed 18 feet apart on the roadway actuated the device. Results were recorded manually.

Description of Test Sections

(a) Suburban Section. A study was conducted on a level, tangent section of the Princes Highway (East) in the vicinity of Harrisfield, State of Victoria, Australia. This section of highway is located within the metropolitan area of Melbourne and, apart from its function as a major radial route, forms part of the interstate highway system.

The test section comprised a 26-foot-wide pavement with a center stripe and was flanked by excellently maintained seven-foot-wide shoulders. There were no traffic control devices over the test section and speeds were limited by the maximum prima facie speed limit of 50 m.p.h.

(b) Urban Section. Here the study was conducted on a level and gently winding highway located in Yarra Park and distant about one mile from the central business district of the City of Melbourne, State of Victoria, Australia. There were no connecting roads, nor were there any traffic control devices, and speeds were controlled by a maximum speed limit of 35 m.p.h. The road consisted of a 40-foot-wide pavement between curbs with a single centerline stripe. Parking is not permitted during periods of heavy flow.

Summary of Results of Field Work

Observations on each of the test sections covered a complete range of traffic conditions from free moving traffic with low densities to periods of peak congestion found during the field studies.

Table II-II sets out the average spot speed, mean difference in speed between successive vehicles and the average travel speeds corresponding to hourly traffic volumes for each of the test sections. Traffic volumes are within fifty vehicles of the hourly figures indicated.

It is to be noted that the travel speeds are generally less than the corresponding spot speeds, the difference being greater in the case of the suburban section than in the urban test section.

1 Freeborn and Whiting, "An Instrument for Measuring the Speeds of Vehicles," Research Note No. 1145, Road Research Laboratory, Great Britain.
# QUALITY AND THEORY OF TRAFFIC FLOW

## TABLE II-II
### SUMMARY OF FIELD DATA

Suburban Test Section -- Princes Highway East

<table>
<thead>
<tr>
<th>Total Volume ± 50 veh.</th>
<th>Average Spot Speed m.p.h.</th>
<th>Mean Diff. in Speeds Between Vehicles m.p.h.</th>
<th>Average Travel Speed m.p.h.</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>45.9</td>
<td>8.4</td>
<td>45.3</td>
</tr>
<tr>
<td>400</td>
<td>41.6</td>
<td>7.0</td>
<td>41.7</td>
</tr>
<tr>
<td>600</td>
<td>40.6</td>
<td>3.7</td>
<td>36.9</td>
</tr>
<tr>
<td>700</td>
<td>37.4</td>
<td>4.2</td>
<td>34.3</td>
</tr>
<tr>
<td>800</td>
<td>34.7</td>
<td>4.6</td>
<td>37.1</td>
</tr>
<tr>
<td>900</td>
<td>36.5</td>
<td>4.2</td>
<td>33.0</td>
</tr>
<tr>
<td>1,000</td>
<td>32.2</td>
<td>3.3</td>
<td>35.1</td>
</tr>
<tr>
<td>1,100</td>
<td>33.0</td>
<td>4.1</td>
<td>33.4</td>
</tr>
<tr>
<td>1,200</td>
<td>33.3</td>
<td>3.0</td>
<td>30.3</td>
</tr>
<tr>
<td>1,300</td>
<td>32.4</td>
<td>2.3</td>
<td>29.8</td>
</tr>
<tr>
<td>1,600</td>
<td>32.1</td>
<td>2.4</td>
<td>29.2</td>
</tr>
</tbody>
</table>

**NOTE:** Volumes are total volumes for both directions of the two-lane, two-way road.
## TABLE II - II
(continued)
SUMMARY OF FIELD DATA

<table>
<thead>
<tr>
<th>Total Volume ± 25 veh. v.p.h.</th>
<th>Average Spot Speed m.p.h.</th>
<th>Mean Diff. in Speeds Between Vehicles m.p.h.</th>
<th>Average Travel Speed m.p.h.</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>--</td>
<td>--</td>
<td>40.7</td>
</tr>
<tr>
<td>650</td>
<td>35.0</td>
<td>4.2</td>
<td>36.5</td>
</tr>
<tr>
<td>700</td>
<td>35.6</td>
<td>3.4</td>
<td>34.6</td>
</tr>
<tr>
<td>750</td>
<td>--</td>
<td>--</td>
<td>34.5</td>
</tr>
<tr>
<td>800</td>
<td>34.2</td>
<td>3.5</td>
<td>33.5</td>
</tr>
<tr>
<td>900</td>
<td>33.4</td>
<td>2.8</td>
<td>33.4</td>
</tr>
<tr>
<td>1,000</td>
<td>--</td>
<td>--</td>
<td>30.1</td>
</tr>
<tr>
<td>1,100</td>
<td>34.3</td>
<td>2.9</td>
<td>--</td>
</tr>
<tr>
<td>1,200</td>
<td>--</td>
<td>--</td>
<td>29.5</td>
</tr>
</tbody>
</table>

NOTE: Volumes shown are volumes for two lanes, one direction of a four-lane, two-way road. Field measurements were made over four lanes and relationships in table are based on an assumed 50-50 directional volume split.
Chapter II - III
ANALYSIS OF DATA

Relation Between Speed Distributions and Volume

The wide range in volumes encountered during this field study covering a limited time did not produce a sufficient number of speed samples at many levels of volume to give statistically significant results. Average speeds were obtained for the different volume levels by assuming a linear relationship between speed and volume and fitting the curve to those points where the data on volume and speed were adequate. The following regression equations resulted:

Princes Highway: \[ S = 44.5 - 1.03 \cdot V_1 \quad r = -0.91 \]
Brunton Avenue: \[ S = 44.9 - 1.27 \cdot V_2 \quad r = -0.90 \]

Where: \[ S \] = mean speed in miles per hour
\[ V_1 \] = volume in hundreds of veh. per hour (both lanes -- both directions of two-lane road)
\[ V_2 \] = volume in hundreds of veh. per hour (two lanes -- one direction of four-lane road)
\[ r \] = coefficient of correlation

Having established a method of estimating the average speed, it becomes possible to set up a family of curves of speed distribution for varying traffic volumes following a technique developed by the author. ¹

Figures II-2 and II-3 illustrate the established relationship of speed distribution for varying traffic volumes for each of the test sections. The purpose of preparing these sets of curves for the present study was to provide a means of estimating coefficients of variation of speeds corresponding to varying volumes.

Briefly, the technique used in establishing these curves depends on the fact that speed distributions closely approximate normal or Gaussian distributions. Such distributions, when plotted in the form of cumulative percentage curves on arithmetic probability paper, take the form of straight lines. Moreover, the slopes of these lines can be estimated with reasonable accuracy from the knowledge of the parameters which characterize

¹H. P. George, "Traffic Behaviour and Road Capacity Study," Research Memorandum No. 12, Country Roads Board, Victoria, Australia, pp. 11, 12.
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the distributions. These parameters are the standard deviation and the average speed, the latter being predictable.

The family of curves is developed by assuming distributions for two widely separated volumes such as free moving and possible capacity, and by a knowledge of the average speeds which are found on the 50 percentile ordinate.

Evaluation of Congestion Ratings for the Test Sections

In Tables II-III and II-IV, a complete range of Congestion Ratings has been computed for each test section, from free moving conditions to the threshold of possible capacity. Densities are determined from the volume and the average speed. The discomfort component has been determined from the density and the standard deviation corresponding to the distributions shown on Figures II-2 and II-3, the standard deviation being estimated from the range in speed between the 7 and 93 percentiles.

In practice, this technique would not be followed as a sufficient number of observations of speed would be taken to give statistically significant estimates of the coefficient of variation. This study, however, is establishing a principle, and the method adopted becomes necessary because of the limited amount of field data collected.
FIGURE II-2
SPEED VS. VOLUME — PRINCES HIGHWAY
## TABLE II - III
EVALUATION OF CONGESTION RATINGS: SUBURBAN TEST SECTION — TWO-LANE, TWO-WAY HIGHWAY

(Princess Highway East, Harrisfield, Victoria)

<table>
<thead>
<tr>
<th>Total Volume (v.p.h.)</th>
<th>Average Speed (m.p.h.)</th>
<th>Coeff. of Variation Per Cent (v)</th>
<th>Average Density (v.p.m.)</th>
<th>Discomfort Factor $\sqrt{0.3v}$ Density</th>
<th>Congestion Ratings Total Both Lanes (veh. hours/mile /hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Travel</td>
<td>Spot</td>
<td>Travel</td>
<td>Spot</td>
<td>Travel</td>
</tr>
<tr>
<td>200</td>
<td>45.3</td>
<td>42.4</td>
<td>17.9</td>
<td>4.4</td>
<td>4.8</td>
</tr>
<tr>
<td>400</td>
<td>41.7</td>
<td>40.4</td>
<td>17.0</td>
<td>9.6</td>
<td>10.0</td>
</tr>
<tr>
<td>600</td>
<td>36.9</td>
<td>38.3</td>
<td>15.7</td>
<td>16.2</td>
<td>15.6</td>
</tr>
<tr>
<td>800</td>
<td>37.1</td>
<td>36.3</td>
<td>14.8</td>
<td>21.4</td>
<td>22.0</td>
</tr>
<tr>
<td>1,000</td>
<td>35.1</td>
<td>34.2</td>
<td>13.6</td>
<td>28.4</td>
<td>29.2</td>
</tr>
<tr>
<td>1,200</td>
<td>30.3</td>
<td>32.1</td>
<td>12.0</td>
<td>39.6</td>
<td>37.4</td>
</tr>
<tr>
<td>1,400</td>
<td>29.5</td>
<td>30.1</td>
<td>10.5</td>
<td>47.4</td>
<td>46.6</td>
</tr>
<tr>
<td>1,600</td>
<td>29.2</td>
<td>28.0</td>
<td>8.72</td>
<td>54.8</td>
<td>57.6</td>
</tr>
<tr>
<td>1,800</td>
<td>25.0</td>
<td>26.0</td>
<td>6.65</td>
<td>72.0</td>
<td>69.2</td>
</tr>
<tr>
<td>*1,900</td>
<td>24.0</td>
<td>25.0</td>
<td>5.15</td>
<td>79.0</td>
<td>76.0</td>
</tr>
</tbody>
</table>

*1,900 v.p.h. is the volume closest to possible capacity.

NOTE: Values of those Spot Speeds used in this table which were not observed are computed from the linear regression equation: $S = 44.5 - 1.03V \cdots \cdots \cdots \cdots \cdots (r = -0.911)$

Values of those travel speeds not observed are computed from the linear regression equation: $S = 45.1 - 1.11V \cdots \cdots \cdots \cdots (r = -0.905)$ where in each case $V =$ volume in hundreds.
FIGURE II-3
SPEED VS. VOLUME — BRUNTON AVENUE
<table>
<thead>
<tr>
<th>One-Way Volume</th>
<th>Average Travel Speed (m.p.h.)</th>
<th>Coeff. of Variation Per Cent (v)</th>
<th>Average Density Per Lane (v.p.m.)</th>
<th>Discomfort Factor 0.3 Density √0.3 v</th>
<th>Congestion Ratings Per Lane (veh.-hrs./mile/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>41.0</td>
<td>13.3</td>
<td>4.9</td>
<td>0.7</td>
<td>5.6</td>
</tr>
<tr>
<td>600</td>
<td>40.7</td>
<td>13.0</td>
<td>7.2</td>
<td>1.1</td>
<td>8.3</td>
</tr>
<tr>
<td>800</td>
<td>33.5</td>
<td>12.6</td>
<td>11.6</td>
<td>1.8</td>
<td>13.4</td>
</tr>
<tr>
<td>1,000</td>
<td>30.1</td>
<td>12.0</td>
<td>16.6</td>
<td>2.6</td>
<td>19.4</td>
</tr>
<tr>
<td>1,200</td>
<td>29.5</td>
<td>11.4</td>
<td>20.3</td>
<td>3.2</td>
<td>23.5</td>
</tr>
<tr>
<td>1,400</td>
<td>27.1</td>
<td>10.5</td>
<td>25.8</td>
<td>4.4</td>
<td>30.2</td>
</tr>
<tr>
<td>1,600</td>
<td>24.5</td>
<td>9.4</td>
<td>32.6</td>
<td>5.9</td>
<td>38.5</td>
</tr>
<tr>
<td>1,800</td>
<td>22.0</td>
<td>8.0</td>
<td>40.8</td>
<td>7.9</td>
<td>48.7</td>
</tr>
<tr>
<td>2,000</td>
<td>19.5</td>
<td>6.4</td>
<td>51.3</td>
<td>11.1</td>
<td>62.4</td>
</tr>
<tr>
<td>* 2,100</td>
<td>18.2</td>
<td>5.9</td>
<td>57.5</td>
<td>13.0</td>
<td>70.5</td>
</tr>
</tbody>
</table>

* 2,100 v.p.h. in one direction is the volume closest to possible capacity.

NOTE: Values of those speeds used in this table which were not observed are computed from the linear regression equation:

\[ S = 44.9 - 1.27V \ldots \ldots \ldots \ldots \quad (r = -0.90) \]

where \( V \) = volume in hundreds
Chapter II-IV
MEASUREMENT OF TRAFFIC INEFFICIENCY

It is obvious that the congestion rating in itself will not give a measure of the inefficiency of traffic operation, except to indicate how inefficiency increases with volume. Some basic or satisfactory condition must be introduced in order to determine the extent of the inefficiency. In addition, due regard has to be paid to the duration of congested flow as well as its intensity during an average day.

Hourly Variations in Volume and Congestion

The relation between total hourly volumes and corresponding congestion ratings for each of the test sections are shown in Figures II-4 and II-5. Figures II-6 and II-7 show the hourly variations in total volumes for both test sections, and the corresponding hour-by-hour congestion ratings are also plotted.

Measurement of Total Daily Inefficiency

In the case of the suburban, two-lane, two-way section, a satisfactory operating condition is assumed to coincide with the practical capacity which has been found to be approximately 600 v.p.h. with an average travel speed of 37 m.p.h. From figure II-4 this corresponds to a rating of 17.5 vehicle-hours per mile per hour. This rating is shown as a dashed line in Figure II-6 and is termed "Satisfactory Service."

The area between the congestion-rating curve and the Satisfactory Service line (shown hatched in Figure II-6) is a measure of the total daily inefficiency. It amounts to 55 vehicle-hours per mile per day.

For the urban four-lane, unsubdivided section, a satisfactory congestion rating of 38 vehicle-hours per lane per mile per hour has been adopted. This corresponds to about 500 v.p.h. per lane with an average speed of 30 m.p.h.

The total inefficiency for the urban highway tested -- relative to the assumed satisfactory operating condition -- turns out to be 252 vehicle-hours per mile per day and is shown in Figure II-7.

A more precise method of determining the daily inefficiency of a section of highway would be to compute the inefficiency in each direction separately, and then add in order to arrive at the
FIGURE II-4
CONGESTION RATING VS. VOLUME — PRINCES HIGHWAY

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FIGURE II-5
CONGESTION RATING VS. VOLUME — BRUNTON AVENUE
total inefficiency. This method would take into account unbalanced flow at peak periods, and additional traffic lanes in the direction of heavier flow. Then proper weight would be given to congested flow. It would probably be the correct procedure in many urban streets.

Traffic Inefficiency Indices

The values determined for total daily inefficiency -- relative to the assumed satisfactory conditions of traffic operation -- may be used as indices for comparison purposes. They also can be used to estimate the cost of the traffic inefficiency by assigning appropriate cost value to the indices.
Chapter II - V
SUMMARY AND CONCLUSIONS

Attempts have been made in this study to measure and evaluate various levels of congestion relative to traffic volumes, average speeds and the distribution of speeds as characterized by the coefficient of variation. This has been done on the premise that time losses and the restriction in freedom of movement constitute the major factors contributing to inefficient traffic operation.

The field work and its analysis were carried out to illustrate the principle involved in a possible approach to a system for rating, in an objective manner, the quality of service afforded by traffic facilities. The analysis also demonstrated the way in which congestion ratings might be used to evaluate the total inefficiency of traffic operation considered in relation to the intensity and duration of congestion found during an average day.
Travel Time Relationships

by

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Mr. Guerin was recipient of the 1958
Annual Alumni Award presented to the
Outstanding Student of the Year by
the Yale Highway Traffic Alumni Association

A Thesis Submitted in Partial Fulfillment of
Requirements for a Certificate in Highway Traffic

BUREAU OF HIGHWAY TRAFFIC
YALE UNIVERSITY
May 1958
Chapter III - I
INTRODUCTION

Of all the variables in traffic stream quality characteristics, travel time is the most easily visualized. This study considers travel time as one of the basic variables in the characteristics of a traffic stream, with particular emphasis on the travel-time density relationship.

Outstanding works on relationships between travel time, density, volume and speed are the studies by O. K. Normann\textsuperscript{1,2} and the studies by J. G. Wardrop.\textsuperscript{3} The earlier works were inspired by an interest in road capacity. For this, volume was considered as the independent variable and speed the dependent variable. The aim was to determine a design volume above which speeds would drop causing unreasonable delay or restriction. Curve 1(a) of Figure III-1 illustrates the bent line by which this relationship has often been expressed in the British literature. Curve 1(b) illustrates the dual curve often produced in American literature with the upper arm labeled "free flowing," and the lower arm labeled "congested." Both arms are curves fitted to points representing speed observations in spite of the fact that at least two publications\textsuperscript{4,5} are careful to point out that observed speeds may fall anywhere between these two arms. This situation would occur if traffic conditions were changing during a period of speed observations at one location. A typical case would be a recording made in advance of a signalized intersection where the flow may be alternately congested and free flowing as the signal shows red and green. When considering average speeds over a length of road, including or adjacent to a signalized intersection, the alternate conditions will occur along the length of the section as well.

QUALITY AND THEORY OF TRAFFIC FLOW

FIGURE III-1
SPEED-VOLUME AND TRAVEL TIME-VOLUME RELATIONSHIPS AS FOUND IN CURRENT LITERATURE

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TRAVEL TIME RELATIONSHIPS

as in time, and the resulting points will lie between the curves.

This leads to the concept that boundary curves are more likely to describe the speed-volume relationships than are the usual average curves. This is of greatest importance when a section of urban road is being considered because of the inevitable variation in traffic conditions along its length.

Since travel time is an inversion of average speed (provided that average speed is appropriately defined) the above discussion will apply equally to it, and the travel-time volume relationship may also be considered as a boundary curve. Curve 2(a) of Figure III-1 illustrates the curve plotted by Rothrock and Keefer,\(^6\) and curve 2(b), as illustrated by Walker,\(^7\) is an inversion of curve 1(b).

Both the travel-time volume and speed-volume curves have the characteristic that there are two values of the dependent variable for most volumes to be considered.

Two relationships that are of particular value in relating time, density, volume and speed have been developed by Wardrop\(^8\) and by Rothrock and Keefer.\(^9\)

Wardrop developed analytically that:

\[
\text{Volume} = \text{Density} \times \text{Space Mean Speed}
\]

One of the basic assumptions in this analysis is that the speed of each vehicle is constant even though speeds vary from vehicle to vehicle. This assumption is not valid in studies of relatively short lengths of road under urban conditions.

The expression used by Rothrock and Keefer:

\[
\text{Density} = \text{Volume} \times \text{Average Travel Time}
\]

is used extensively in the Analysis of Data section of this study.

Both of the above relationships present the difficulty of determining what is the applicable value of volume when conditions are changing. The concepts of input volume and output volume are discussed and used in Chapter III, Analysis of Data.

The conventional curves, 1(b) and 2(b) of Figure III-1, have been used to construct curves 3, 4 and 5 of Figure III-2 with density as the independent variable. The upper arm of the speed-volume curve was constructed as the best straight-line correla-

\(^7\) W. P. Walker, \textit{op. cit.}, p. 38.
\(^8\) J. G. Wardrop, \textit{op. cit.}, p. 330.
QUALITY AND THEORY OF TRAFFIC FLOW

tion of the data observed and may be better represented by a curve. If so, the derived curves will be modified and smoothed somewhat.

The most certain way of finding the true relationship between any two of these values is to plot the directly observed values of each. Density has rarely been used as the independent variable, in spite of the advantages indicated by the form of curves 3, 4 and 5 of Figure III-2. A discussion of methods available for measuring density is therefore warranted.

Density is a variable which has been difficult to measure by direct instrumentation. In short sections, the number of vehicles is generally counted at regular intervals either by direct vision or from successive frames of photographs. Streb\textsuperscript{10} and Solomon\textsuperscript{11} have investigated the accuracy of such methods for various intervals between observations.

On longer sections, a running record of input and output enables density at any instant to be computed provided the initial density is known. Because they may be applied to longer sections, methods described in this study may represent a significant improvement in previous methods of observing density.


TRAVEL TIME RELATIONSHIPS

(1) Speed-Volume.

(2) Travel Time-Volume.

(3) Speed-Density.

(4) Travel Time-Density.

(5) Volume-Density.

(6) Travel Time-Speed.

FIGURE III-2
TYPICAL RELATIONSHIP CURVES AS DERIVED FROM NORMANN'S
SPEED-VOLUME CURVE

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Chapter III - II
LOCATIONS AND METHODS OF STUDY

Washington Boulevard, Chicago, Illinois

The field work for this location was designed and performed by the Chicago Area Transportation Study as a speed-density study. It was conducted by L. E. Keefer, Traffic Engineer, using a method suggested by R. L. Creighton, Assistant Study Director.¹

The site is a quarter-mile section of Washington Boulevard between Menard Avenue and Austin Boulevard in the City of Chicago. The section consists of three full blocks with a fixed-time signal at the western end. A uniform width of 40 feet, undivided, with parking removed on the north side from 4:00 p.m. to 6:00 p.m., provides two moving traffic lanes for the heavy westbound commuter movement. Only this westbound movement was studied. There is no commercial traffic except buses, and these make stops in each block. A sketch plan is shown in Figure III-3.

The study was made from 4:30 p.m. to 6:00 p.m. on Friday, March 8, 1957. The road was dry and the weather clear.

Observers A and C made manual cumulative counts of entering and leaving vehicles at each end of the section. The observers' watches were synchronized and the cumulative counts were noted at the end of each one-minute period. The purpose was to keep a running record of the number of vehicles in the traffic stream between observers A and C at any time. A third observer, B, recorded vehicles entering or leaving the section between A and C. At the end of the test period the observers made a check on the cumulative counts.

Traffic during the period of study ranged from moderate to heavy, but the congestion during the heaviest traffic periods was not so severe as to cause a breakdown of traffic flow.

The analysis made of the data from this study is original, the purpose being different from the analysis of the Chicago Area Transportation Study made on the same data.

Temple Street, New Haven, Connecticut

The field work for this location was designed and conducted

TRAVEL TIME RELATIONSHIPS

(1) Study Site, Washington Boulevard, Chicago, Ill.

(2) Study Site, Temple Street, New Haven, Conn.

FIGURE III-3
SKETCH PLANS OF SECTIONS STUDIED

(Not to scale)
QUALITY AND THEORY OF TRAFFIC FLOW

by the author. The basic tool was a desk calculator which was used to accumulate vehicle time-of-occupancy as it was being observed.

The site was a 460-foot section of Temple Street, running southwest from Trumbull Street to a bridge over the N.Y.N.H. & H. Railroad branch track. The section has no intersecting streets. The width tapers from 39 feet to 36 feet and parking is permitted (and is well utilized) on both sides, providing two lanes of one-way, city-bound traffic. The flow included approximately 3% buses and 3% trucks during the periods of observation. The buses do not stop within the section. A sketch is shown in Figure III-3.

Care was taken in the selection of the site to avoid parallax errors caused by observing from an acute angle. The entry to the section was the crosswalk at the Trumbull Street intersection -- the far side from the point of view of traffic and the near side from the point of view of the observer. The exit from the section was the abutment of the bridge over the railroad tracks. The configuration of both was such as to reduce parallax errors in judging the precise moment of entry or exit.

The major factor which this site satisfies is that speeds of vehicles vary in all possible ways:

(a) vary from vehicle to vehicle
(b) vary along the length of the route
(c) vary at any one point with changing phases of the signal at the approach

With this variation, it may be expected that any type of relationship found between the variables on this site will have general application.

Study No. 1 at this location was performed from 2:15 p.m. to 4:45 p.m. on Thursday, February 20, 1958. Heavy snow had fallen during the previous week end and had been plowed into windrows occupying five feet of roadway on each side. There was still parking as usual on both sides leaving room for only one lane of moving traffic. The weather was clear and the central portion of the pavement was dry. Snow had been removed from the Grove Street Exit permitting the intersection approach to operate under normal conditions.

Study No. 2 at this location was made from 2:00 p.m. to 5:00 p.m. on Thursday, March 13, 1958. The weather was cloudy but dry, and the whole width of pavement was free of snow, dry and available for two lanes of moving traffic plus the usual parking.
The whole section was visible from the third-floor, bay window of an adjacent building where the four observers were located with an electric desk calculator. Observers A and B operated the calculator to measure the input and output, respectively. Observer C recorded the data from the calculator at the end of each test. Observer D observed and recorded the number of vehicles in the section at the start and end of each test, the number of parking and unparking activities, entries and exits from private driveways, the number of buses and trucks and any relevant circumstances which affected the flow. The operation of the calculator is of some interest and will be described in detail.

Starting with a cleared machine, operator B commenced the test by pressing the plus bar and holding it down. The top register immediately recorded the passage of time at the rate of 588 digits per minute. As the first vehicle entered the section, operator A depressed No. 1 key in the second column from the left side of the keyboard. The machine was operated in the repeat position so this key stayed in position until key No. 2 directly above it was depressed as the second vehicle came into the section. Meanwhile operator B recorded out-going vehicles in a similar manner on the two columns of keys at the right side of the keyboard. The test ended as soon as operator B released the plus bar, and neither operator then touched the machine until the following were recorded:

(a) the keys depressed on the left side of the keyboard, representing the number of vehicles which had entered the section during the test
(b) the keys depressed on the right side of the keyboard similarly representing output volume
(c) the upper register, representing the duration of the test
(d) the left side of the lower register representing the time-accumulation of input vehicles
(e) the right side of the lower register representing the time-accumulation of output vehicles.

The tests were run to coincide with the phases of the signal at the entry end of the test section, and to give short-duration data under a variety of conditions:

(a) red phase only
(b) green phase only
(c) one cycle commencing with green phase
(d) one cycle commencing with red phase
QUALITY AND THEORY OF TRAFFIC FLOW

(e) two cycles commencing with green phase
(f) two cycles commencing with red phase

Traffic during the period of the study No. 1 ranged from light to very heavy. During the heavy traffic, the section almost filled with vehicles at each green phase of the traffic signal near the entry point. About 500 feet beyond the exit point was another signalized intersection which had a shorter green phase causing the vehicles to back up and proceed intermittently in accordian fashion.

Traffic during study No. 2 was light to moderate. Some mild congestion occurred during four of the tests as vehicles queued for the signal ahead of the intersection.
Chapter III - III
ANALYSIS OF DATA

Washington Boulevard Study

From 4:30 p.m. to 5:59 p.m., 2,115 vehicles were counted into the section and 1,988 were counted out at the end of the section. At intermediate points, 23 vehicles were counted in and 149 counted out.

The following information is readily obtained by differences from the cumulative counts for any time which is an exact minute \( t \) during the period of observation:

(a) Instantaneous density at time \( t \).
(b) Input volume for one minute \((t - 1)\) to \( t \).
(c) Output volume for one minute \((t - 1)\) to \( t \).

The question now arises as to which densities correspond to which volumes. Owing to the difference in the times to which they apply, it is apparent that the values (a), (b) and (c), above, cannot be related without some combinations. The simplest way is to combine many values so that the effects of end discrepancies will be negligible, but this process submerges many short-term variations which may be of considerable interest.

A method of combining density values and volume values so that they may be correlated was required. Calculations of travel time indicated a range of from one to three minutes, with an average of about one and one-half minutes. Those over two minutes occurred together from about 5:42 p.m. to 5:55 p.m. To bring together the times of occurrence of the density values and volume values, an even number of volumes were related with an odd number of densities.

For the biggest part of the period, when travel times were less than two minutes, the following relationships were adopted. They seemed to be the best expression of the concepts of average volume and average density during the travel time of any slug of traffic.

Average density at time \( t = \frac{1}{4} (\text{density at time } t - 1 \]
\[+ 2 \times \text{density at time } t\]
\[+ \text{density at time } t + 1)\]
QUALITY AND THEORY OF TRAFFIC FLOW

Average volume at time $t = \frac{1}{4}$ (input volume over period $t - 2, t - 1$
+ input volume over period $t - 1, t$
+ output volume over period $t, t + 1$
+ output volume over period $t + 1, t + 2$)

When the travel time is greater than two minutes, not all the densities and volumes which comprise any slug of traffic are taken into account by the above method of averaging. To avoid such discrepancies the following relationships are adopted for twelve consecutive minutes in which travel times are between two and three minutes:

Average density at time $t = \frac{1}{8}$ (density at time $t - 2$
+ $2 \times$ density at time $t - 1$
+ $2 \times$ density at time $t$
+ $2 \times$ density at time $t + 1$
+ density at time $t + 2$)

Average volume at time $t = \frac{1}{8}$ (input volume over period $t - 3, t - 2$
+ $2 \times$ input volume over period $t - 2, t - 1$
+ input volume over period $t - 1, t$
+ output volume over period $t, t + 1$
+ $2 \times$ output volume over period $t + 1, t + 2$
+ output volume over period $t + 2, t + 3$)

The effect of vehicles leaving the section at intermediate points must be considered. Since vehicles which leave the section are counted out during the interval in which they leave, the density value always represents the true number of vehicles in the section. However, the intermediate counts do not appear in the volume formula. A vehicle leaving by a side street is counted only as part of the input volume and therefore has only half of the effect that a through vehicle has on the average volume. This is considered to be reasonable.

Using the above formulae, average volumes and average densities are found for 83 consecutive values of $t$. Volume and density are plotted in Figure III-6. Density was chosen as the independent variable for the reasons discussed in Chapter III-I of
TRAVEL TIME RELATIONSHIPS

This study. The correlation between volume and density was poor, the linear correlation analysis giving $r^2 = 0.043$. This was to be expected since there was a mixture of free flowing and congested conditions, and the typical curves of Figure III-1 indicate that a straight-line relationship is not to be expected under these conditions.

Travel time was computed for each of the 83 values of $t$ using the formula quoted in Chapter III-I:

$$\text{Density} = \text{Volume} \times \text{Average Travel Time}$$

Plotting travel time against density, Figure III-4, gave a very good linear correlation of $r^2 = 0.885$; i.e., 88.5% of the variation in travel time can be attributed to a linear relationship with density.

The typical curves of Figure III-1 indicate a relationship which could be close to linear for the range of conditions found. Plotting travel time against volume, Figure III-5, gave such a scatter of points that the linear correlation was judged by inspection to be negligible.

Temple Street Study No. 1

The data were first examined to ensure that the number of vehicles checked balanced out as follows:

- Number of vehicles entering section during test
- Number of vehicles in traffic stream at start of test
- Number of vehicles unparking during test
- Number of vehicles leaving section during test
- Number of vehicles in traffic stream at end of test
- Number of vehicles parking during test.

The few vehicles which entered or left the section at private driveways were classified as unparking or parking, respectively, as the effect on the traffic stream was similar. Six tests were rejected because of the failure of the number of vehicles to check.

The duration of each test was readily computed using the speed of the machine, 588 digits per minute:

$$\text{Duration of test (minutes)} = \frac{\text{upper register}}{588}$$

The traffic volume applicable to each test was computed:

$$\text{Volume (vehicles per minute)} = \frac{\text{Number of vehicles entering} + \text{Number of vehicles leaving}}{2 \times \text{Duration of test}}$$

Two-lane, One-way Traffic Stream.

\[ T = 0.080 + 0.0389K \]

\[ r^2 = 0.885 \]

\[ T = \frac{0.003K^2}{\sqrt{128 - K}} + 0.70 \]

**FIGURE III-4**
TRAVEL TIME-DENSITY RELATIONSHIP — WASHINGTON BOULEVARD
TRAVEL TIME RELATIONSHIPS

Two-lane, One-way Traffic Stream.

\[ T = \frac{0.003 Q^2 T^2}{\sqrt{128 - QT}} + 0.70 \]

Volume \((Q)\), vehicles per minute.

FIGURE III-5
TRAVEL TIME-VOLUME RELATIONSHIP — WASHINGTON BOULEVARD
FIGURE III-6
VOLUME-DENSITY RELATIONSHIP — WASHINGTON BOULEVARD

\[ Q = \frac{16.57 + 0.169 K}{r^2} = 0.043 \]

Two-lane, One-way Traffic Stream.
TRAVEL TIME RELATIONSHIPS

This formula gives an effect of one-half vehicle to any vehicle which parks, unparks or is in the section at the start or end of the test. This is considered reasonable unless all the particular activity of one of these types occurs at one end of the section. This did occur when short-duration tests were made using only the red phase of the signal. Six more tests were rejected because of this, leaving 35 tests out of the total of 47 tests made.

The traffic density applicable to each test was computed using the principle of vehicle time-of-occupancy:

\[
\text{Density (vehicles per 460-ft. section)} = \frac{\text{Number of vehicles in traffic stream at start of test}}{\text{Upper register}} + \frac{\text{Left side lower register - Right side lower register}}{\text{Upper register}} + \frac{1}{2} \text{number of vehicles which unpark during test}
\]

The last term gives an average effect to those vehicles which come into the traffic stream by unparking. It could be argued that there should be a similar negative term for those vehicles which park during the test. However, these vehicles take a considerable time to maneuver into parking position and thus contribute to the density of the traffic stream for more than their moving time, so the negative term was omitted. There are weaknesses in the adopted procedure as affected by parking and unparking, but the number of such maneuvers is small and should not unduly affect the results.

Travel time was computed for each test using the formula:

\[\text{Density} = \text{Volume} \times \text{Average Travel Time}\]

Travel time is plotted against density in Figure III-7, and shows a very high linear correlation \((r^2 = 0.856)\). Travel time against volume, Figure III-8, and volume against density, Figure III-9, show negligible linear correlation.

Temple Street Study No. 2

The data were treated in an identical manner to that used in Temple Street Study No. 1. There were 39 usable tests.

Travel time plotted against density in Figure III-10 gives a good linear correlation \((r^2 = 0.648)\). Travel time plotted against volume, Figure III-11, gives a low linear correlation \((r^2 = 0.068)\). These results are similar to those obtained in the two previous tests. Volume plotted against density, Figure III-12, gives a good linear correlation \((r^2 = 0.541)\) which was unexpected in view of the low correlations for this combination in the two previous tests. There were less points relating to congested conditions in this test; effect of this will be discussed in the next section.
One-lane, One-way Traffic stream.

\[ T = 0.046 + 0.118K \]

\[ r^2 = 0.856 \]

\[ T = \frac{0.015 \cdot K^2}{\sqrt{23 - K}} + 0.20 \]

**FIGURE III-7**
TRAVEL TIME-DENSITY RELATIONSHIP — TEMPLE STREET STUDY NO. 1

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TRAVEL TIME RELATIONSHIPS

One-lane, One-way Traffic Stream.

\[ T = \frac{0.015 Q^2 T^2}{\sqrt{23 - QT}} + 0.20 \]

FIGURE III-8
TRAVEL TIME-VOLUME RELATIONSHIP — TEMPLE STREET STUDY NO. 1
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One-lane, One-way Traffic Stream.

\[ Q = \frac{K}{0.015 K^2 \sqrt{23 - K}} + 0.2 \]

Volume \( Q \), vehicles per minute.

Density \( K \), vehicles per 460 feet.

FIGURE III-9
VOLUME-DENSITY RELATIONSHIP — TEMPLE STREET STUDY NO. 1

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TRAVEL TIME RELATIONSHIPS

Two-lane, One-way Traffic Stream.

\[ T = 0.143 + 0.0416K \]
\[ r^2 = 0.648 \]

\[ T = \frac{0.026 K^2}{\sqrt{46 - K}} + 0.14 \]

Density (K), vehicles per 460 feet.

FIGURE III-10
TRAVEL TIME-DENSITY RELATIONSHIP — TEMPLE STREET STUDY NO. 2
QUALITY AND THEORY OF TRAFFIC FLOW

\[
T = 0.193 + 0.007Q
\]

\[
r^2 = 0.068
\]

Two-lane, One-way Traffic Stream.

\[
T = \frac{0.026 Q^2 T^2}{\sqrt{46 - QT}} + 0.14
\]

Volume (C), vehicles per minute.

Travel Time (T), minutes per 460 feet.

FIGURE III-11
TRAVEL TIME-VOLUME RELATIONSHIP — TEMPLE STREET STUDY NO.2

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\[ Q = \frac{K}{0.026K^2 + 0.14} + 1.804K \]

\[ r^2 = 0.641 \]

Two-lane, One-way Traffic Stream.

FIGURE III-12
VOLUME-DENSITY RELATIONSHIP — TEMPLE STREET STUDY NO. 2
QUALITY AND THEORY OF TRAFFIC FLOW

Discussion of Results

Some of the linear correlations which have been obtained in these three tests are very high and could easily lead one to the conclusion that straight-line relationships existed between the variables. Such a conclusion would not be valid because, as most statistical textbooks will point out, the linear correlation is merely the best fitting straight line and does not predict the relationship unless it is known from other sources that it is linear.

Some simple logic concerning stream characteristics will reveal that none of these relationships can be linear. Consider a two-lane, one-way traffic stream. At very low volumes and densities a driver will be free to choose his own speed except when an overtaking maneuver ahead of him blocks his freedom to overtake. In very light traffic, the frequency of such occurrences will be approximately proportional to the square of either the density or the volume. This indicates that curves of speed or travel time plotted against volume or density will have properties similar to a quadratic equation when both volume and density are low; i.e., when the slope of the curve is zero and the curvature constant.

At the point of maximum volume, there is no logical reason to believe that there is some magical speed represented by a sharply pointed graph, especially when we are considering the general case of a finite length of roadway along which conditions are varying. White has analyzed this phenomenon and shown it to be analogous to electrical wave theory. Small variations in speed can modulate to have a large effect, but a modulation at one point on a roadway could be altered by varying speed conditions along the route under consideration. The overall effect will be a smooth curve at the turning point representing maximum volume. A sharp-ended curve may be due to an over-emphasis of the terms "free flowing" and "congested." In this study, they are considered to be relative terms with no sharp dividing line between them.

The lower arm of the speed-volume curve also has some relationships which can be logically deduced. Because of the relationship,

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Volume = Density \times Speed

and the physical limitation of bumper-to-bumper conditions on density, the speed-volume ratio cannot be less than the reciprocal of maximum density. As speed increases, drivers will make space gaps greater than those under bumper-to-bumper conditions. Thus, the slope of the speed-volume curve will be the reciprocal of maximum density at the zero point, and will curve above and away from this finite slope.

Many formulae were tried in an attempt to find one which satisfied all the above conditions and which was simple to visualize. The one which met all requirements is mostly expressed in terms of travel time, \( T \), and density, \( K \):

\[
T = \frac{aK^2}{\sqrt{b - K}} + c
\]

Where:  
\( a \) is a constant such that \( 2a \) is equal to the curvature of the time-density curve at zero density.  
\( b \) is a constant equal to the maximum density (bumper-to-bumper conditions)  
\( c \) is a constant equal to the minimum travel time

The boundary curves shown on Figures III-4 to III-12 were plotted from the above formula, with constants chosen for each of the three tests to fit the relevant data. The relationships between other pairs of variables were found from the time-density formula using the known relationship:

\[
Density = Volume \times Travel Time
\]

To illustrate the way in which the formula fits the requirements of the speed-volume relationship, Figure III-13 was plotted using the constants derived from the Washington Boulevard data. Speed was defined as the reciprocal of average travel time; i.e., it may be regarded as a space-mean operating speed.

The units used in each study are consistent so that the simple relationships between volume, density, speed and time will prevail. In the Washington Boulevard study the units are:

- Travel Time: Minutes
- Density (two-lane): Vehicles Per 1/4 Mile
- Volume (two-lane): Vehicles Per Minute
- Speed: 1/4 Miles Per Minute

The formulae quoted for linear correlation and for the boundary curves are quoted in terms of these units. Similarly in the
FIGURE III-13
SPEED-VOLUME RELATIONSHIP — WASHINGTON BOULEVARD STUDY
TRAVEL TIME RELATIONSHIPS

Temple Street studies the units are:

<table>
<thead>
<tr>
<th>Travel Time</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>Vehicles Per 460 Feet</td>
</tr>
<tr>
<td>Volume</td>
<td>Vehicles Per Minute</td>
</tr>
<tr>
<td>Speed</td>
<td>460 Feet Per Minute</td>
</tr>
</tbody>
</table>

In Temple Street Study No. 1, densities and volumes are one-lane, and in Study No. 2, they are two-lane.

The boundary curves have many features which confirm that the shapes adopted are reasonable. The travel time-density curve starts with zero slope and commences to curve upwards away from the minimum travel time at a rate proportional to the square of the density. It rises to unlimited travel times as the density approaches asymptotically the line of maximum density representing complete congestion. The travel time-volume curve starts similarly but turns smoothly as the maximum volume (or capacity) is reached and then rises to unlimited travel times as the volume approaches zero, asymptotically, again representing complete congestion. The volume-density curve rises from zero at a slope equal to the speed of free flowing traffic, and curves smoothly at the maximum volume. The width near the top of the curve represents the range of density which may occur between practical and possible capacity. The curve drops off as congestion increases, then flattens a little before dropping rapidly to zero at maximum density. This illustrates the conditions under which small volumes may be maintained under strictly queued conditions. These features are common to the curves for all three studies, but are best illustrated in Figures III-7, III-8 and III-9. The data used in these curves covered the wide range of conditions which were observed in Temple Street Study No. 1.

A further check on the reasonableness of the results is to compare the maximum values of the volume curves with the possible capacities of the sections as determined by the procedures of the Highway Capacity Manual. For Washington Boulevard, the curve shows 1,875 vehicles per hour, and the computed possible capacity is 2,200 vehicles per hour when considered as uninterrupted flow, or 900 vehicles per hour when considered on the basis of signal capacity. The Temple Street Study No. 1, when the width was restricted by windrows of snow, indicates a capacity of 1,100 vehicles per hour. The computed possible capacity as an uninterrupted one-way street is 1,550 vehicles per hour, but the capacity of the signalized intersection at Grove

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\(^5\) Highway Capacity Manual, Committee on Highway Capacity, Highway Research Board, 1950, Part V.
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Street is only 640 vehicles per hour. Similarly, the Temple Street Study No. 2 under snow-free conditions indicates 1,260 vehicles per hour compared with an uninterrupted possible capacity of 3,000 vehicles per hour and a signal capacity of 640 vehicles per hour. In all three studies, the indicated capacity is more than the capacity of the signal controlling the exit, and less than the capacity of the section considered as an uninterrupted route. This is reasonable because the method assesses the characteristics of the whole length of the section and not just its weakest point.

Before accepting boundary curves as the best description of the data, it is well to have another look at the linear relationships. Travel time-density gave three good correlations with \( r^2 \) values of 0.885, 0.856 and 0.648. The last one, illustrated in Figure III-10, is lower because of three relatively high travel times during periods of low density. Such values will be found at random intervals and may be caused by such a simple matter as a car cruising to find a parking place. Owing to the small periods of observation (0.6 to 2.6 minutes), such causes will have noticeable effects. In fact, the reason for choosing small periods of observation was to prevent these effects from being submerged in averages. Chance occurrences and short-term variations in the traffic stream are all represented in the plotted points. Travel time-volume gave negligible linear correlations on all three studies, but the volume-density correlations were interesting giving \( r^2 \) values of 0.043 and 0.641 for the two which were computed. The other was judged by inspection to be negligible. The high value of 0.641, illustrated in Figure III-12, was examined and it was found that the major contributing factors were those same points which lowered the travel time-density correlation. These low volume points helped balance the line and guide it towards three other points representing high density conditions. If all six of these points were omitted, the regression line would have a slope nearly double that of the present line.

Linear relationships based on correlation analyses are seen to be affected greatly by chance occurrences and by variations in the range of operating conditions during the period of the study.
Chapter III - IV
UNSOLVED PROBLEMS

Researchers who have studied travel time-volume relationships have adopted various forms of curves to represent their findings. Straight-line relationships are used by some including Volk and Prager even though they recognize that a straight-line relationship does not satisfy all conditions. Volk points out that the correlations and equations given are at best indicative of a possible relationship and are not based on sufficient data. Prager was determining formulae for use in mathematical models of traffic flow in a network. The use of a straight-line relationship here is justified on the grounds that the computations are simplified.

An improvement on straight-line relationships is the curving of the upper end of the line to indicate the rapid increase in travel time as volume approaches capacity. Further advanced are the completely curved relationships derived by Newell and Webster.

For the speed-volume relationship, many researchers have been content to adopt Normann's form of curve (as discussed in Chapter III-I) with a straight-line relationship for the upper arm of the curve. A different approach was adopted by Huber who used a parabola with a horizontal axis, derived by converting a straight-line relationship between speed and density. This approach had been used earlier by Greenshields.

Even the best of these relationships have some defects when examined critically, and some do not have the compensating advantage of simplicity of application. Some of the requirements

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discussed in Chapter III-III are:

(a) Curves of speed or travel time against volume or density will have zero slope and constant curvature at zero density and volume.

(b) The lower arm of the speed-volume curve will have a slope equal to the reciprocal of the maximum density at zero speed and volume.

(c) Curves will be smooth throughout as there is no sharp division between "free flowing" and "congested" conditions.

(d) The travel time-volume curve has two values of travel time for each volume.

The author does not suggest that the relationships developed in this study have overcome all difficulties, but can think of many more which need to be overcome before the methods used here could be accepted without reserve.

The most fundamental difficulty is that the variables are of different basic types. Time and speed are continuous variables, but both density and volume have the discrete variable of an integral number of vehicles as their numerators. This is generally overlooked when one is dealing with large numbers of vehicles, but creates difficulties when small slugs of traffic are considered. The methods used to match volumes with their corresponding densities in Chapter III-III overcame the immediate difficulty, but are not satisfying from a fundamental point of view.

The nature of the relationships between variables has not been firmly established. Good linear correlations have been found between almost every pair of variables, but it is obvious that not more than one pair (and properly none) can have a true linear relationship. Seeking a better way of expressing the relationships, the author has emphasized that they should be expressed as boundary curves rather than as average curves. If this idea is accepted, the use of such curves for practical traffic engineering problems must be considered. It is of little use to know that a function may be limited by a certain line unless it is known how close to the line it is likely to be; i.e., the probability of the occurrence of travel times less than a chosen travel time at some specified volume or density.

The possibility of using boundary curves to plot probabilities is illustrated in Figure III-14. The boundary curve could be regarded not as an absolute limit, but as a limit which is broken by, say, 1% of observations. This would have a stabilizing effect on the boundary curve because rare observations of exceptionally
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PERCENTAGE INDICATED IS PERCENTAGE OF OBSERVATIONS OF TRAVEL TIME WHICH ARE EXPECTED TO BE BELOW THESE CURVES.

FIGURE III-14
USE OF BOUNDARY CURVES TO ESTIMATE PROBABILITIES OF TRAVEL TIME
QUALITY AND THEORY OF TRAFFIC FLOW

high or low travel times would not cause unwarranted changes in the curve. Figure III-14 shows probability lines as they would appear if sufficient data were available to plot them. The ranges of probable values of travel time are smaller when plotted against density than when plotted against volume. At moderate to heavy volumes on urban streets, travel time may increase considerably as density increases and yet have little effect on the volumes which leave the section. At these times input volumes will, of course, exceed the output volumes. These factors emphasize that density would be a better independent variable than volume.

In order to use density as an independent variable in practical traffic engineering problems, better methods are needed to measure and estimate it. One approach would be to use Origin and Destination data to predict the number of vehicles which would be in an arterial street during any time period, particularly at peak periods for commuter traffic. Similarly, the number of vehicles on the roads in a whole area such as the Central Business District could be estimated and the overall effect on travel times of any additional density could be estimated. Such a process would have many applications in highway planning in estimating the overall effect of increased motor vehicle ownership. An illustration of the use of this broad approach is seen in Gru-now's study\(^1\) of delays at intersections on an annual basis.

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Chapter III - V
CONCLUSIONS

The relationships between travel time, speed, volume and density can be shown graphically by curved forms. Results which are expressed as linear relationships are unreliable because the values of the regression coefficients depend on the range of traffic conditions observed and on the duration of the observations.

The relationships may be expressed better by boundary curves than by average curves. Boundary curves of the form

\[ T = \frac{aK^2}{\sqrt{b - K}} + c \]

Where: \( T \) is travel time
\( K \) is density, and
\( a, b \) and \( c \) are constants

satisfy all the requirements which have been considered.

Density appears to be the most satisfactory parameter to use as the independent variable in travel time investigations.
The Development of Traffic Congestion

by

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A Thesis Submitted in Partial Fulfillment of Requirements for a Certificate in Highway Traffic

BUREAU OF HIGHWAY TRAFFIC
YALE UNIVERSITY
May 1959
Chapter IV - I

INTRODUCTION

This study is concerned with the behaviour of drivers in a traffic stream as the flow (or volume) in the stream approaches the absolute capacity of the highway. Previous studies have shown that as traffic increases it reaches a point where the operating conditions and speeds start to degenerate quickly and the whole flow may slow down almost to a halt, thus greatly reducing the road’s carrying capacity and increasing operation costs and accident hazards.

Several different mathematical models describing traffic behaviour under congested and near-congested conditions have been proposed. The following analysis was undertaken to test the validity of some of these models and to obtain a fuller description of the actual traffic behaviour at the point of congestion. The analysis was particularly aimed at investigating Lighthill and Whitham’s Kinematic Flow Theory.¹

This study analyzes in detail the pattern of traffic behaviour on the Merritt Parkway during a thirty-minute period. The traffic data were collected in 1956 on the approach side of a pair of temporary Bailey bridges at a point where a parkway culvert had been washed out by severe floods.² During the study period the character of traffic flow changed from relatively free flow to almost “stop and go” operation with a long queue of vehicles.

Definitions

The following terms are used extensively in this report. To avoid confusion they are defined:

Concentration - k: Often called “density.”

The concentration of a stream of traffic is the number of vehicles on a fixed length of roadway at a given time. It is denoted by the letter "k" and often expressed in terms of vehicles per mile, "v.p.m."


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Flow - q: Often called "volume."
The flow of a stream of traffic is the number of vehicles passing a fixed point within a given period of time. It is denoted by the letter "q" and usually expressed in terms of vehicles per hour, "v.p.h."

Space Mean Speed - v
The arithmetic mean speed of all the vehicles on a given length of roadway at the same instance.
Or ... the harmonic mean speed of all vehicles passing a point in a given time period.
Or ... the speed calculated from the average time vehicles take to traverse a given length of roadway. In this study, this speed is denoted by the letter "v."

Background
There have been several previous practical and theoretical attempts to define the relationships between traffic speed, concentration and flow. The results of some of these are closely related to this study and, therefore, a brief outline of the main conclusions previously reached is included below. This is in logical, rather than historical, order.

1. J. G. Wardrop 3 considered a normal traffic flow to be a combination of a number of uniform traffic streams traveling at constant speeds. Each flow had its own speed, v, flow, q, and concentration, k. He then proved that:

\[ q = k \cdot v \]

Where:
\[ q = \text{sum of } q_i, \quad k = \text{sum of } k_i \text{ and } v = \text{Space Mean Speed} \]

Wardrop also presented some survey results showing a straight-line relationship between q and mean running speed on central London streets. This covered a range of running speeds of from 24 m.p.h. to 18 m.p.h. and in some cases down as far as 10 m.p.h.

2. B. D. Greenshields 4 postulated (on the basis of fairly limited data) a straight-line relationship between concentration, k, and speed. This result can be transformed by Wardrop's formula to

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Shape found by Road Research Laboratory, G.B.

Speed $v$. (miles/hour)

Concentration $k$. (veh/mile)

Greenshields, (basic form - linear)

Greenberg (basic form - exponential)

---

Flow $q$. (veh/hour)

Greenshields (derived)

Greenberg (derived)

---

O.K. Normann, Highway Capacity Manual (basic form - linear)

---

Concentration $k$. (veh/mile)

Greenshields (derived)

Greenberg (derived - very similar to form used by Lighthill and Whitham)

---

FIGURE IV-1
THEORETICAL RELATIONSHIPS BETWEEN FLOW, CONCENTRATION AND SPEED

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QUALITY AND THEORY OF TRAFFIC FLOW

give parabolic curves for both $q$ against $k$ and $v$ against $q$. The maximum $q$ therefore occurs at the mid-range of both $k$ and $v$. If the range of $v$ is 0 to 60 m.p.h. then the maximum $q$ will occur at 30 m.p.h. This transformation assumes that the speed given by Greenshields is Space Mean Speed. The shapes of the $q$:k and $v$:k curves obtained in this way are very sensitive to any deviation of the original $v$:k curve from a straight line.

The family of three curves generated in this way is shown in Figure IV-1, together with some of the other suggested relations between $v$, $q$ and $k$.

3. O. K. Normann\(^5\) found a straight-line relationship between average speed and flow, $q$, within the range 30 to 50 m.p.h. on rural highways. This relation was incorporated in the Highway Capacity Manual\(^6\) in 1950 with the addition of a dotted, curved line running from the end of the straight $q$:v line back to the origin. The Manual postulates that once the $q$:v relation has dropped from the solid line to the dotted line it cannot return to the solid line (relatively free flow) until $q$ is less than the maximum rate of flow from a stationary queue of vehicles (stated as about 1,500 vehs. per lane per hour). For conditions on the solid line above this value of $q$, the flow could be said to be "supersaturated" and rather unstable.

Normann's relationship is plotted on the $q$:v graph in the center of Figure IV-1.

4. M. J. Huber\(^7\) applied the straight-line relation between $k$ and $v$ postulated by Greenshields to data collected on the Merritt Parkway in the vicinity of a temporary bridge. (These data are the same as those on which this thesis is based.) Space mean speed, $v$, and concentration, $k$, were calculated for a series of five-minute-time slices and a straight-line correlation of .96 to .97 was found between the two. The line obtained corresponded to a maximum $q$ at 18 to 19 m.p.h.

5. H. Greenberg\(^8\) used the same data and some of his own collected from the Lincoln Tunnels under the Hudson River. He found that in both cases the $k$, $v$ values obtained fitted a relation

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\(^7\) M. J. Huber, op. cit.

THE DEVELOPMENT OF TRAFFIC CONGESTION

THE MEANING OF A $q,k$ (Flow: Concentration) CURVE

Sample flow condition, $k=50$ vpm, $q=1600$ vph, $v=50$ mph
Wave Velocity = Tangent Slope = 10 mph.

Maximum Rate of Flow = 1750 vph/lane
at 90 vpm and 20 mph.

$50$ mph Vector
Free Running Speed

Traffic Behaviour Near a Bottleneck

$q,k$ Curve for highway
$q,k$ Curve for Bottleneck

Flow 1.
Flow 2.
Flow 3.

FIGURE IV.2
THE MEANING OF A $q,k$ (FLOW: CONCENTRATION) CURVE
TRAFFIC BEHAVIOUR NEAR A BOTTLENECK

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QUALITY AND THEORY OF TRAFFIC FLOW

of the form: \( k = a \cdot e^{-bv} \) (where \( a \) and \( b \) are constants) better than a straight line. The effect of this hypothesis on the shape of the \( v:q \) and \( q:k \) curves is shown in Figure IV-1.

6. M. J. Lighthill and G. B. Whitham\(^9\) have developed theoretically an explanation of the behaviour of traffic "waves" on long, crowded roads and when traffic flows through a bottleneck. Their theory is based on the existence of a smooth, reversible \( q:k \) curve and does not allow for the "supersaturated" traffic condition suggested by O. K. Normann.

Figure IV-2 illustrates this theory. The upper diagram of Figure IV-2 is a hypothetical \( q:k \) curve -- starting off from the origin as a straight line and then curving as it rises to a maximum \( q \) value, then decreasing smoothly to zero as \( k \) increases to a point corresponding to the concentration of a stopped queue of vehicles.

As traffic conditions change they are assumed to follow the \( q:k \) curve and from the graph the concentration of any given flow can be predicted. The \( q:k \) curve would change in magnitude and possibly in shape with weather conditions and also with changes in road cross-section, clearances and sight distances, etc. The slope of a line on the graph represents speed:

\[
\frac{dq}{dk} = \frac{\text{Vehs.}}{\text{Time}} \cdot \frac{\text{Distance}}{\text{Vehs.}} = \frac{\text{Distance}}{\text{Time}} = \text{Speed}
\]

The slope of the initial straight section of the \( q:k \) curve is the free-flowing speed of the facility, the speed observed at low traffic flows. Then as traffic conditions progress around the curve, the traffic speed is given by the slope of the ray from the origin to the appropriate point on the \( q:k \) curve.

Traffic "waves" are parts of the traffic stream where vehicles are bunched together more closely than the average stream spacings. These bunchings do not always contain the same vehicles, but move along the traffic stream so that different vehicles are included in the wave at different times. The velocity of one of these waves with respect to the ground is given by the slope of the tangent to the \( q:k \) curve at the \( q:k \) point corresponding to the flow and concentration conditions. Traffic shock waves are formed where two different traffic concentrations meet; e.g., at the point where a smooth flowing stream of traffic meets a stationary queue of vehicles. These shock waves travel at the speed given by the slope of the chord joining the two \( q:k \) points representing the operating conditions of the two concentrations.

\(^9\) M. J. Lighthill and G. B. Whitham, op. cit.
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These slopes may be either positive or negative and, thus, the waves may travel either forward or backward with respect to the ground. The slope of a chord or tangent, however, is always less than the slope of the ray from the origin. Therefore, the speed of one of these waves is always less than the speed of the traffic, and the waves always move backward and never forward with respect to the vehicles in the traffic stream. Before the point of maximum flow, the waves move forward with respect to the ground. After this point, they change direction and move backward. The velocity of a starting wave in a stopped queue of vehicles is given by the slope of the tangent at the point where the curve cuts the concentration axis.

The second part of Figure IV-2 illustrates the effect of increasing traffic flow in the vicinity of a traffic bottleneck. The q:k curves are shown, the upper one for the highway in advance of the bottleneck and the lower one for the bottleneck itself. As the flow increases from flow 1 to flow 2, the traffic concentration on the road preceding the bottleneck follows the top curve. It rises almost linearly and the speed changes very little. The concentration close to the bottleneck follows the lower curve and increases much more rapidly while the speed starts to decrease. Then as the flow rises to level 3, the speed in the bottleneck drops rapidly, and the flow is limited to the maximum q on the lower curve (this is the point x). If more flow than this arrives, it accumulates as a queue in advance of the bottleneck, and the traffic conditions in this area change from those of point y on the top q:k curve to those of point z.

This change can happen quite suddenly, and it involves an abrupt drop in speed and rise in concentration with little change in flow. The traffic in advance of the bottleneck will then be traveling slower than the traffic in the bottleneck, and it will also be spaced closer together. The traffic conditions will remain like this until the flow drops below level 3 and the queue has been discharged through the bottleneck. As soon as the tail of the queue clears a point in advance of the bottleneck, the q:k relation at that point will return to the left side of the upper q:k curve and traffic will resume a higher speed.

7. L. C. Edie and R. S. Foote, compared the predictions of Lighthill and Whitham's paper with the traffic data they had collected on both sides of the traffic tunnels under the Hudson River.

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QUALITY AND THEORY OF TRAFFIC FLOW

Their results were in general agreement with the theory when analyzed in one-minute periods, but the points observed upstream of the bottleneck during congested flow tended to lie along a horizontal line rather than cluster around a single point on what would be the right-hand side of the q:k curve. The exact location of the bottleneck in the tunnel was not known, and a linear relation between v and k was assumed.

Purpose

This thesis is an attempt to investigate in more detail the variations in q, k and v in the vicinity of a known bottleneck. The results are mostly in the form of graphs of the variables in terms of each other, and the resulting traces obtained are compared with these previous studies.
Chapter IV- II
FIELD STUDY

The data on which this study is based were obtained by the Connecticut State Highway Department and the Bureau of Highway Traffic, Yale University. The data were recorded at Norwalk, Connecticut, on the Merritt Parkway, a major, four-lane, divided and limited-access highway between New York and New England. The parkway carries only passenger vehicles. The study site was on the approach side of two temporary Bailey Bridges (one in each of the two lanes) spanning a pavement washout which had been caused by severe floods the previous year. The bridges were used by westbound vehicles traveling toward New York. The study was carried out on a warm Sunday afternoon on a partly cloudy-to-sunny summer day. As this parkway is a popular recreational route, the traffic volume became very heavy, and during the study traffic started to form a queue in advance of the bridges.

The traffic analyzed was that in the median lane of the highway, and its behaviour was recorded with an Esterline-Angus 20-pen recorder. Six pneumatic tube detectors were coupled to the recorder and located as shown in Figure IV-3. This figure also shows the highway profile and the temporary ramp used to raise traffic 24 inches to the deck of the Bailey Bridge. The photograph at the top of the figure shows the bridge as it appeared to an approaching driver about 200 feet away.

Both lanes of the highway were 13-feet wide. There were no sealed shoulders, but on either side of the 26-foot roadway there were low mountable curbs flanked by an 8-foot grass shoulder on the right side and a 22-foot grass median on the left side. From the recorder tube at point e to the bridge, 24-inch traffic cones were placed along the lane line to separate the two lanes. The bridge in the median lane was 80 feet long, 14 feet, 6 inches between trusses and paved over 12 feet, 6 inches of this. The left-hand truss was three feet high and the right-hand truss was eight feet high.

The study was carried out on Sunday, July 8, 1956, and this analysis covers the period from 5:05 p.m. to 5:30 p.m. only. At approximately 5:20 p.m. the traffic speed through the area decreased noticeably due to traffic congestion originating at the
bridge and substantial queues of traffic formed. Unfortunately, because of difficulties in operating the recorder, the record for the two center tubes, c and d, is not complete, and it has been possible to use this in only one part of the analysis. The data used in this analysis has been used only as a volume record in the previously published results of this study¹, and none of the data collected at this site has been analyzed in this way previously.

The normal speed limit on the Parkway is 55 m.p.h., but for 4,000 feet in advance of the temporary bridge a 25 m.p.h. speed limit had been imposed because of the temporary bridges. In addition, several signs warned drivers of the "Temporary Bridge Ahead" and directed them to "Stay in Line." Preliminary observations of traffic during periods of low concentration showed that the minimum speeds of traffic and the greatest concentration appeared to occur some distance in advance of the temporary bridge. Accordingly, the recording apparatus was set up to cover this part of the roadway.

¹ M. J. Huber, op. cit.
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TEMPORARY BRIDGE SITE
(As seen by approaching traffic.)

PROFILE OF STUDY SITE

Direction of Traffic Flow

Recorder Tubes

80 ft. long bridge
100 ft. ramp to raise traffic 24 ins. to bridge.

Horizontal Scale: 100 ft to an inch.
Vertical Scale: 10 ft to an inch.

FIGURE IV-3

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Chapter IV - III
ANALYSIS

The 25 minutes of data collected were analyzed in different ways in an attempt to discover significant patterns of results. Of these studies, the most significant are illustrated and described in this section of the paper. The analyses covered are as follows:

(a) Analysis by Minute Periods
(b) Analysis by Groups of Vehicles
(c) Analysis by Individual Vehicles
(d) Relation Between Spacing and Speed
(e) Travel Time
(f) Vehicle Speed Patterns

In previous analyses of flow, concentration and speed, the most common type of analysis has been to divide the data into equal time slices and then compute the flow, the average concentration and the space mean speed for each time slice. This has been done for slices of one hour 30 minutes, five minutes and, recently, for one-minute periods. The results of these analyses have led to the general shapes of the curves described in the "Background" section of this paper. It is possible for considerable changes of flow and concentration to occur within the longer time slices, and it is practically impossible to get samples of the transition stage between low and high concentration flows with time slices of this magnitude. This difficulty leads to uncertainty as to the shape of the behaviour curves at their midpoints in the transition area; e.g., few points have been found in the center of the v:k curve, and thus it is difficult to determine whether this curve is straight or curved. The maximum possible value of q and the values of k and v at which this should occur are seriously affected as shown in Figure IV-1. Therefore, it is very desirable to be able to use short time slices to define the shape of the relationship curves. Traffic, however, is actually a series of random events rather than a continuous flow.

Hypotheses such as Wardrop's $q = k \cdot v$, and Lighthill and Whitham's Kinematic Wave Theory which are based on fluid analogies, are true only when averaged over reasonably long periods or distances. This condition is due mainly to the difficulty of relating $q$ and $k$ as these are not measured on common bases.
THE DEVELOPMENT OF TRAFFIC CONGESTION

"q" has reference to a point on the roadway and the arrivals at this point must be averaged with time. "k" has reference to a fixed length of highway and has an integral value for every instant of time.

If the length of highway is too short, k becomes very erratic. This difficulty can be overcome by averaging the values of k with time, but if this is done k becomes purely theoretical and cannot be observed directly. A k calculated in this way over a very short distance is essentially:

\[
k = \frac{\text{Sum of Travel Times}}{\text{Total Time Period}} \times \frac{\text{Number of Vehicles}}{\text{Number of Vehicles}}
\]

Then
\[
k = \frac{\text{Sum of Travel Times}}{\text{Number of Vehicles}} \times \frac{\text{Number of Vehicles}}{\text{Total Time Period}}
\]

or \[k = \frac{q}{v}\]

Thus, Wardrop's equation is exactly true if k is measured in this way. The following analyses use both this relation and some independent measures of k and attempt to find how far the fluid analogies remain valid as the lengths of time slices are reduced.

Analysis by Minute Periods

The first analysis divides the data into time slices of one minute. For each minute, the flow q is computed from the number of vehicles passing tube a (see plan of study site, Figure IV-3, page 118). The concentration is computed independently by averaging the number of vehicles between tubes a and e at 5, 15, 25, 35, 45 and 55 seconds after the start of each minute. Speeds are calculated by computing the space mean speed of all the vehicles passing tube a during the minute. This was done on the basis of travel times over each of the three distances, tube a to tube b, tube e to tube f, and tube a to tube f. "q," "k" and "v" are, therefore, independently measured and so cannot agree exactly as to reference times and road lengths.

Figure IV-4 shows on the upper graph the variations in flow and concentration during the study period. It will be noticed that the flow is erratic until about 5:18 p.m., but from then on it becomes relatively steady at a slightly higher level than it had reached before. The concentration follows the general undulations of the flow curve, but at about 5:19 p.m. it rises suddenly from a level of 30 vehicles per mile to about 100 vehicles per
QUALITY AND THEORY OF TRAFFIC FLOW

TRAFFIC FLOW AND CONCENTRATION

SPEED

FIGURE IV-4
THE DEVELOPMENT OF TRAFFIC CONGESTION

mile. It then remains fairly level at this new height.

The lower portion of Figure IV-4 shows the speed records for the traps a-b and e-f. The trap closest to the bridge, e-f, is a fairly faithful mirror image of q and k until 5:18 p.m. when it drops from about 20 m.p.h. to about 12 m.p.h. and then remains steady at this speed. Trap a-b gives a higher initial speed -- about 35 m.p.h. -- but suddenly at 5:20 p.m. it drops to 8 m.p.h. and then remains level until the end of the study.

This data shows that a relatively small increase of traffic about 5:18 p.m. led to a very great change in operating conditions on the road within the next two minutes. A traffic queue started to form in advance of the bridge. Within two minutes this had reached back to trap e-f nearly 500 feet in advance of the bridge. A speed reduction of 8 m.p.h. close to the bridge resulted in a drop of 27 m.p.h. 500 feet in advance of the bridge. The traffic at this advance point began to move even more slowly than the traffic negotiating the bridge itself. This result is very similar to that predicted by Lighthill and Whitham's theory, and it shows that acute congestion can occur very quickly.

Figure IV-5 refers to the minute analysis and shows a set of v:k, v:q and q:k diagrams plotted from the q and k in each minute (measured as described above) and the v derived from travel times between tubes a and f over the whole length of the study area. The points are labelled to show the minutes they represent. In each case, point 5:19 p.m. is a transition stage between all the points before this and all the points after.

The v:k points appear to lie on a curve of the general shape suggested by Greenberg and the scatter here is quite small. The v:q and q:k points are more scattered and could be consistent with most of the theories except that the points representing congested traffic have an average q of about the highest q observed in the free flowing points. It might be expected that some free flowing points would have had a greater q than the congested points, but the shape obtained is consistent with the pattern given by Lighthill and Whitham for traffic in a bottleneck.

The v:k points are much more consistent than the v:q and q:k points. This suggests that there is a large fluctuation in the measure of q used. It might be possible to obtain more consistent v:q and q:k curves if q were measured at some other point or in some other way. Table IV-1 shows the data from which these curves were plotted, and in the right hand column estimates q from the values of k and v. There is fair agreement between the measured and estimated values of q.
QUALITY AND THEORY OF TRAFFIC FLOW

FIGURE IV-5
MINUTE ANALYSIS
## THE DEVELOPMENT OF TRAFFIC CONGESTION

### TABLE IV - I

**MINUTE ANALYSIS OF FLOW, CONCENTRATION AND SPEED**

<table>
<thead>
<tr>
<th>Minute starting</th>
<th>Vehicles passing' in a minute</th>
<th>Ave. no. of vehs. in section</th>
<th>Average Transit time (sec)</th>
<th>Rate &quot;q&quot; vehicles per hour</th>
<th>Concent. &quot;k&quot; vehs/mile (S.M.S.)</th>
<th>Speed &quot;v&quot; (m.p.h)</th>
<th>Estimate of &quot;q&quot; = k.v</th>
</tr>
</thead>
<tbody>
<tr>
<td>5:06 pm</td>
<td>10</td>
<td>1.3</td>
<td>9.3</td>
<td>600</td>
<td>15</td>
<td>35</td>
<td>530</td>
</tr>
<tr>
<td>5:07 pm</td>
<td>13</td>
<td>2.2</td>
<td>11.2</td>
<td>780</td>
<td>24</td>
<td>29</td>
<td>700</td>
</tr>
<tr>
<td>5:08 pm</td>
<td>17</td>
<td>3.2</td>
<td>10.8</td>
<td>1020</td>
<td>35</td>
<td>30</td>
<td>1020</td>
</tr>
<tr>
<td>5:09 pm</td>
<td>8</td>
<td>1.3</td>
<td>9.3</td>
<td>480</td>
<td>15</td>
<td>35</td>
<td>530</td>
</tr>
<tr>
<td>5:10 pm</td>
<td>11</td>
<td>1.8</td>
<td>9.3</td>
<td>660</td>
<td>20</td>
<td>35</td>
<td>700</td>
</tr>
<tr>
<td>5:11 pm</td>
<td>14</td>
<td>1.8</td>
<td>10.9</td>
<td>840</td>
<td>20</td>
<td>30</td>
<td>600</td>
</tr>
<tr>
<td>5:12 pm</td>
<td>18</td>
<td>3.8</td>
<td>13.5</td>
<td>1080</td>
<td>42</td>
<td>24</td>
<td>1000</td>
</tr>
<tr>
<td>5:13 pm</td>
<td>14</td>
<td>3.5</td>
<td>15.1</td>
<td>840</td>
<td>39</td>
<td>21</td>
<td>820</td>
</tr>
<tr>
<td>5:14 pm</td>
<td>15</td>
<td>2.0</td>
<td>10.7</td>
<td>900</td>
<td>22</td>
<td>30</td>
<td>660</td>
</tr>
<tr>
<td>5:15 pm</td>
<td>10</td>
<td>2.3</td>
<td>10.9</td>
<td>600</td>
<td>26</td>
<td>30</td>
<td>780</td>
</tr>
<tr>
<td>5:16 pm</td>
<td>11</td>
<td>1.3</td>
<td>10.6</td>
<td>660</td>
<td>15</td>
<td>31</td>
<td>460</td>
</tr>
<tr>
<td>5:17 pm</td>
<td>16</td>
<td>3.0</td>
<td>10.6</td>
<td>960</td>
<td>33</td>
<td>31</td>
<td>1020</td>
</tr>
<tr>
<td>5:18 pm</td>
<td>16</td>
<td>2.7</td>
<td>11.7</td>
<td>960</td>
<td>30</td>
<td>28</td>
<td>840</td>
</tr>
<tr>
<td>5:19 pm</td>
<td>20</td>
<td>6.0</td>
<td>20</td>
<td>1200</td>
<td>67</td>
<td>16</td>
<td>1040</td>
</tr>
<tr>
<td>5:20 pm</td>
<td>20</td>
<td>9.3</td>
<td>33</td>
<td>1200</td>
<td>103</td>
<td>9.8</td>
<td>1000</td>
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<tr>
<td>5:21 pm</td>
<td>18</td>
<td>9.3</td>
<td>30</td>
<td>1080</td>
<td>103</td>
<td>10.8</td>
<td>1110</td>
</tr>
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<td>10.3</td>
<td>32</td>
<td>1200</td>
<td>115</td>
<td>10.2</td>
<td>1180</td>
</tr>
<tr>
<td>5:23 pm</td>
<td>18</td>
<td>10.5</td>
<td>35</td>
<td>1080</td>
<td>117</td>
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</table>
QUALITY AND THEORY OF TRAFFIC FLOW

Analysis by Groups of Vehicles

This analysis is an attempt to obtain shorter time slices without losing the continuity of the observed variables. Groups of five vehicles were used to obtain approximately 15-second time slices during the period of heavy flow while at the same time retaining a reasonable smoothing effect during quieter periods. Two sets of values were obtained, one for trap a-b and the other for trap e-f. The variables q, k and v were obtained independently as follows:

"q" was estimated by averaging the five time spacings between six successive vehicles and then converting this headway into flow per hour.

"k" was calculated by computing an estimate of k for each of five vehicles and then averaging the five estimates. The estimate used was:

\[ k_1 = \frac{\text{Travel time for front vehicle to cover 50 ft.}}{\text{Time spacing at first tube}} \]

"v" was obtained by averaging the times taken by each of the five vehicles to cover the 50-ft. trap and then converting this to miles per hour.

These are illustrated on a space-time diagram in Figure IV-6.

Figure IV-7 is a plot of the v:k, v:q and q:k relations for the a-b trap. The v:k curve shows a good separation between the points observed before congestion was apparent (all above the 20 m.p.h. line) and the points recorded after this. The path of the points between these two conditions is shown dotted. The points recorded during the period of traffic congestion lie on a well-defined line but the precongested points are quite scattered. Two possible relationship lines are sketched in, one a continuous curved line and the other a pair of lines shown dashed.

On the v:q graph the noncongested points are again clearly separated from the congested points, and the latter are very closely grouped together showing little variation in either v or q. The q:k graph is more scattered. There is a definite tendency for the noncongested points to form a straight line from the origin, and the congested points again tend to be grouped together. Dashed curves have been put through the points on these lower graphs to show how the results would be interpreted by Lighthill and Whitham's hypothesis. The dotted curves, however, are not consistent with the v:k curve if Wardrop's equation between the
Illustrating Methods of obtaining "q", "k" and "v" for Vehicle Group Analysis.

Vehicle No.1 in the front of each group is also vehicle No.6 of the preceding group.

**Formulæ:**

\[ q^n = \frac{3,600}{\sum_{i=1}^{5} \frac{a_i}{t_i}} \]

\[ v^n = \frac{50 \times 0.68}{\sum_{i=1}^{5} \frac{t_i}{t_i}} \]

\[ k^n = \sum_{i=1}^{5} \frac{a_i \times 3,600}{50 \times 0.68} = 21.1 \sum_{i=1}^{5} \frac{a_i}{t_i} \]

**FIGURE IV-6**

**SPACE-TIME DIAGRAM**

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QUALITY AND THEORY OF TRAFFIC FLOW

three variables applies in this case. These three curves represent traffic flow upstream from a traffic bottleneck.

Figure IV-8 shows the same information for the trap e-f close to the temporary bridge. Here the two groups of points, congested and noncongested, are closer together than in the previous figure. In the cases of the v:k and v:q charts it is still possible to draw a line between the two groups, thus dividing them into "before" and "after" categories. The patterns obtained on these graphs are consistent with Lighthill and Whitham's hypothesis regarding traffic conditions in a bottleneck.

If Figures IV-7 and IV-8 are superimposed it will be seen that in each case the points of Figure IV-8 fall between the two groups of points in Figure IV-7. This phenomenon would indicate that, although the highway cross section is the same in both cases, the presence of the temporary bridge and traffic cones, etc., modifies the traffic characteristic curves at the closer trap e-f in a way similar to that illustrated in the lower part of Figure IV-2.

Figures IV-9 and IV-10 show the same graphs computed from groups of nine vehicles. In this case, k was computed from q and v using Wardrop's formula q = k · v. The pattern is clearer in this case, and the points lie closer to their characteristic lines.

Analysis by Individual Vehicles

Figure IV-11 shows v:k and q:k curves based on observations of single vehicles. "k" has an integral value equal to the number of vehicles between tubes a and f at the instant the associated vehicle passed tube a. "v" is based on the travel time between tubes a and f, and "q" is the product of k and v. To obtain the relationship lines, curves were plotted through the median values of v and q for each value of k.

Again there is a well-defined transition zone between the noncongested and the congested points with only two or three points located in this zone. All readings above and to the left of the transition zone occurred before 5:19 p.m. + 30 secs. Except for four points near the zone, all readings below and to the right occurred after 5:19 p.m. + 45 secs. Then, by 5:20 p.m. + 15 secs., all points were occurring to the right of the line labelled "2" and there were no points to the left of this line until 5:27 p.m.

This is an example of another way of treating traffic characteristics data -- averaging one factor for each value of another. The results are interesting in that they suggest that the v:k curve levels out at about 10 m.p.h. in this instance. Thus, the q:k curve has a saddle shape. This curve, of course, applies only to the combination of highway and bottleneck. There is some
doubt as to the validity of estimating q from k and v in this case as k is an instantaneous measure while v is based on a 10- to 30-second travel time. Both, however, are based on the same road distance and, thus, this approach represents a "space average" measure rather than the "time average" used in the previous graphs.

The congested points are concentrated on the far side of the saddle. This concentration could be expected. On the downhill part of the saddle an increase in k would produce a decrease in q, and this condition is theoretically unstable. The height of the saddle would appear to depend on the capacity of the bottleneck.

Relation Between Spacing and Speed

In the analysis, it became apparent that the speed of a vehicle was conditioned mostly by the speed of the preceding vehicle in a queue, and only to a relatively minor extent by the spacing between the two vehicles. In the same way, the front vehicle is affected by the speed and spacing of the vehicle in front of it, and so on to the front of the queue. When a queue of closely spaced vehicles traveled through the e-f trap close to the temporary bridge, the speeds in the queue tended to decrease until frequently the rear vehicle was traveling at only half the speed of the leading vehicle. Between queues the speeds returned to almost their original level.

The speeds were measured in terms of the time each vehicle took to cover the 50-ft. trap. Figure IV-12 is a plot of the change in this time between two successive vehicles against their time spacing at the beginning of the trap, tube e. Below a time spacing of four seconds, the change in travel time is mostly positive, showing that the rear vehicle was traveling slower than the front vehicle. Above four seconds, the rear vehicle was often traveling faster. This pattern was found to be quite uniform from 35 m.p.h. to nine m.p.h. The average line seems to be level at about +.1 secs. below four-second spacing and then negative at more than four-second spacing.

Suppose, therefore, that a queue of vehicles at two- or three-second spacing meets a bottleneck of this type. If the leading vehicle is traveling at 30 m.p.h., it would take 1.13 seconds to cover 50 feet. The tenth vehicle would take $1.13 + 10 \times .1 \text{ secs.} = 2.13 \text{ secs.}$ to cover the same distance, and would be traveling about 16 m.p.h. On a multilane facility the length of traffic queues is proportional to the traffic concentration, so this relationship will lead again to an inverse relation between v and k as has already been found. However, if some influence on the high-
QUALITY AND THEORY OF TRAFFIC FLOW

FIGURE IV-7
ANALYSIS BY GROUPS OF FIVE VEHICLES
First Trap - Tubes α-β

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FIGURE IV - 8
ANALYSIS BY GROUPS OF FIVE VEHICLES

Last Trap - Tubes e-f

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QUALITY AND THEORY OF TRAFFIC FLOW

\[ \begin{align*}
\text{FIGURE IV-9} \\
\text{ANALYSIS BY GROUPS OF NINE VEHICLES} \\
\text{First Trap - Tubes a-b}
\end{align*} \]
FIGURE IV-10
ANALYSIS BY GROUPS OF NINE VEHICLES

Last Trap - Tubes e-t
QUALITY AND THEORY OF TRAFFIC FLOW

FIGURE IV-11
ANALYSIS BY INDIVIDUAL VEHICLES
THE DEVELOPMENT OF TRAFFIC CONGESTION

Time: 5:06 to 5:21pm, before congestion apparent. Speed range 35mph to 9mph.

Change between Two Successive Vehicles of the time taken to cover the 50 feet between tube e and tube f. Secs.

FIGURE IV-12
RELATION BETWEEN SPACING AND SPEED
Entrance to Bottleneck - Tubes e-f

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way, such as slow moving trucks, tends to reduce passing opportunities and increase the length of traffic queues out of proportion to the increase in concentration, it will tend to have an accelerated effect in depressing speeds through a bottleneck. This effect will still exist even though the trucks themselves have no difficulty in passing through the bottleneck at the speed of the other traffic, and it may be sufficient to trigger off the onset of congestion at lower flows than would otherwise have been the case.

Travel Time

Figure IV-13 illustrates the total travel time between tube a and tube e. This figure is a plot against time of the information used in Figure IV-11 for vehicle speeds, and it reflects the effect of congestion on the driver. Before 5:19 p.m. + 20 secs., travel time over the 427-ft. course was close to 10 secs., but it started to rise and reached a peak at 5:20 p.m. + 40 secs. Then it settled down to a level of about 30 seconds with a trough of 20 seconds about 5:27 p.m. to 5:28 p.m. From an operating cost point of view, it is clear that the development of congestion in this area cost each motorist approximately 20 seconds extra traveling time to cover just over 400 feet. This loss did not permit the road to carry any greater flow of traffic. For this reason, the development of congestion should be avoided if at all possible even at the cost of excluding some vehicles from the road altogether so that the rest may maintain reasonable operating speeds.

Vehicle Speed Patterns

The pattern of vehicular speeds in the traffic stream shows a well-defined wave form as illustrated in Figure IV-14, which is a plot of the time taken by each vehicle to cross each of the traps a-b and e-f. In this figure, the time scale represents the time that the vehicle passed tube a, and the speed records for the same vehicle at both traps are plotted in the same vertical line. By comparing the two curves, it is easy to see which vehicles slowed down and which vehicles increased their speeds. Before 5:21 p.m. + 20 secs., all vehicles were slowing down between the two traps as they approached the bottleneck. After that time, almost all vehicles were traveling faster at the close trap e-f than at the distant trap a-b.

The reduction in speed at both points as congestion became apparent is also clearly shown on this chart. This reduction developed rapidly at 5:19 p.m. at the close trap, but more gradually between 5:19 p.m. and 5:21 p.m. + 30 secs. at the distant trap. The distant trap, a-b, developed a regular wave pattern.
FIGURE IV-13
TRAVEL TIME
(To cover 477 feet from tube α to tube β)
Reference:

Trap a-b ....
Trap c-f ....

Horizontal Scale: Speeds of 20 vehicles plotted per inch.

FIGURE IV-14
TIME FOR VEHICLES TO COVER FIFTY FEET

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FIGURE IV-15
PASSAGE TIME OF VEHICLE WHEELBASES OVER DETECTOR TUBES
QUALITY AND THEORY OF TRAFFIC FLOW

of travel times after 5:20 p.m. The waves represent a fluctuation of from seven m.p.h. to 11 m.p.h. and have a period of almost one minute. This wave motion is not evident at the close trap e-f, due to the fact that small waves (too small to be detected on this scale) generated at the bottleneck run together and increase both their period and amplitude as they travel back through the traffic stream.

Figure IV-15 is another means of investigating this wave pattern of speeds. In this case, the time taken for the wheelbase of each car to cross each of the tubes is plotted. There are five plots, one for each of the tubes a, b, e and f; the center plot of the five represents tubes c and d. On this figure, the time scale represents the exact time of each recording.

It will be noticed that some of the sharpest peaks were recorded at c-d and the wave motion at this point seems to be even more marked than at either a-b or e-f. The wave period is just over one minute and the amplitude at c-d is about 12 m.p.h. to five m.p.h. or less.

From these traces it can be seen that the waves travel back through the approaching traffic stream. The speed of this movement, shown by the sloping lines, is 30 m.p.h. This finding is again generally consistent with the hypothesis of Lighthill and Whitham and conforms with their predictions regarding traffic operation in advance of a congested bottleneck.
Chapter IV - IV
CONCLUSIONS

The following conclusions are drawn from this study:

Lighthill and Whitham's Kinematic Flow Theory concerning the flow of traffic on long, crowded roads and the behaviour of a heavy flow as it passes through a highway bottleneck describes fairly accurately the behaviour observed on the Merritt Parkway at the temporary bridge site.

It seems possible from the traces of the q;k curves in the analysis that O. K. Normann's "supersaturated" flow condition does occur for short periods. Under this condition, waves develop in the traffic stream which soon produce temporary stoppages, and thus limit the flow to the rate of departures from a stopped queue, approximating Lighthill and Whitham's smooth q;k curve form. This wave pattern is most evident in vehicular speeds and seems significant enough to warrant further investigation.

Consistent relationship graphs between q, k and v can be obtained by averaging these values for groups of about five to nine vehicles. This gives one point for each 30 seconds during congested and critical periods, but yet maintains a good smoothing effect under light traffic conditions. This method of dividing traffic characteristics data is recommended for future studies.

Queues of vehicles spaced closer than four seconds apart tend to reduce their speeds as they negotiate a bottleneck. The reduction found was equivalent to an increase of one-tenth of a second per vehicle in the time taken to cover 50 feet. The total speed reduction depends on the number of vehicles in a queue so that long queues lead to greater speed reductions.

A relatively small increase in traffic load on an already heavily loaded highway can cause a very great change in the operating conditions for all traffic on the road. The average travel time over quite a distance may triple within a few minutes, and the highway immediately loses very much of its efficiency. It is desirable to avoid this "congested" condition even if it involves denying some vehicles the use of the highway when the flow reaches a critical level. To facilitate this and as an aid in locating and removing potential causes of bottlenecks, e.g., disabled vehicles,
QUALITY AND THEORY OF TRAFFIC FLOW

a traffic monitoring device to sense the development of congestion would be very valuable. Such a device should give the traffic controlling authority almost immediate warning of the development of congestion and could also be made to illuminate signs and divert traffic automatically.
Speed, Volume and Density Relationships

by

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A Thesis Submitted in Partial Fulfillment of
Requirements for a Certificate in Highway Traffic

BUREAU OF HIGHWAY TRAFFIC
YALE UNIVERSITY
May 1960
Field data for two sites are analyzed. It is shown that, over the range of the data, there seems to be an exponential relationship between the space mean speed of traffic and its density. Families of curves, expressing relationships between travel time and density, speed and density, travel time and volume and speed and volume in terms of probabilities are next produced for the two study sites.

The validity of the exponential speed-density model is examined. It is found that it cannot be applied to expressway-type facilities, although it seems to have application to lower type roadways.

A generalized speed-volume diagram is introduced. This diagram consists of three separate zones -- a zone of normal flow, a zone of unstable flow and a zone of forced flow -- each zone being specified in terms of probabilities. The diagram is checked against available speed-volume data for several different facilities, and it is compared with existing flow models. It appears that the diagram has considerable promise. Further development of it could lead to a clearer understanding of traffic flow with consequent benefits in highway design, operation and control.

Notation and Definitions

- **S = Space Mean Speed:** The harmonic mean speed of all vehicles passing a fixed point on a roadway in a given period of time.
- **V = Volume:** The number of vehicles passing a fixed point on a roadway in a given period of time.
- **D = Density:** The number of vehicles on a fixed length of roadway at a given instant.
- **T = Travel Time:** The time taken by a vehicle to travel between two points, a given distance apart, on a roadway.
QUALITY AND THEORY OF TRAFFIC FLOW

Overall Speed: The total distance covered, divided by the total time taken, including the time lost in delays and stops.

Running Speed: The total distance covered, divided by the total time that the vehicle is in motion.

a, b, c, k, constants

e, exponential constant

r, coefficient of correlation
Chapter V - II
ANALYSIS OF DATA

MERRITT PARKWAY

Speed-Density Relationships

The data were subdivided into one-minute intervals. For each one-minute group, the space mean speed and the density were calculated and plotted in Figure V-1. Also plotted in figure V-1 are curves representing the theories of Greenshields, Nor- mann and Greenberg. ¹ These curves were not fitted statistically, but rather their constants were determined by trial and error because, for present purposes, their form is of more interest than is a precise plot.

Inspection indicates that of the three curves, Greenberg's gives the best representation of the trend of the points over the range observed in this study. Outside the range of data, however, it does not fit the boundary conditions, known within reasonable limits, as well as would be desired. For instance, under traffic-jam conditions, with cars stopped bumper-to-bumper, the density should be of the order of 220 vehicles per mile. ² The greatest weakness of Greenberg's equation is that it gives unrealistic speeds for low densities, the speed approaching infinity as the density approaches zero.

Guerin ³ has given some thought to the shape of the speed-volume curve, and has suggested that the simplest equation, satisfying various stream flow characteristics and expressed in terms of the travel time, T, and density, D, is of the form ⁴

\[ T = \frac{aD^2}{b - D} + c \]

¹ Descriptions of these theories were deleted for this publication because they have already been covered by Guerin and Palmer in this symposium.

² Greenberg obtained values of 215 v.p.m. for the Merritt Parkway, and 228 v.p.m. for the Lincoln Tunnel. His Merritt Parkway data came from the same source as that now being analyzed although the two sets of data do not cover the same period of time.

³ See third paper in this symposium.

QUALITY AND THEORY OF TRAFFIC FLOW

Note: These curves are not fitted statistically.

after Greenberg, \[ D = ke \]

after Normann.

after Greenshields, \( S = a - bD \)

FIGURE V-1
SPEED-DENSITY PLOT — MERRITT PARKWAY
where \( a \) is a constant such that \( 2a \) is equal to the curvature of the time density curve at zero density, and \( b \) is a constant equal to the maximum density.

Guerin reasoned that among other properties of a satisfactory model, its speed or travel time against volume or density should have zero slope and constant curvature at low volume and density.\(^5\)

While the above equation, when expressed in terms of speed and density does fit the data in Figure V-1 well, it is not a particularly easy equation with which to work. Accordingly, attempts were made to arrive at an expression which fitted the data as well, but which was more amenable to mathematical treatment. Equations of the form --

\[
S = \frac{d - D}{aD^2 + b}
\]

\[
S = \frac{d^2 - D^2}{aD^2 + b}
\]

fit the data well, but they do not lend themselves to statistical curve fitting.\(^6\) Admittedly, a curve may be fitted to a scatter of points either by a method of successive approximations, or by a combination of inspection and graphical approximation,\(^7\) but such a solution is not as useful as a more rigorous mathematical analysis.

The main requirements of any model could be stated as:

(a) It must fit with a high degree of accuracy all available data.
(b) It should be capable of extrapolation over the full range of values normally found in practice.
(c) Its limitations, if any, should be known.
(d) It should lend itself readily to mathematical analysis.

A curve that does give a good fit over the range of available data and which can be handled mathematically with relative ease, is

\[
S = ae^{-\frac{D}{b}}
\]

where \( a \) and \( b \) are constants.

\(^5\) Ibid., p. 38.

\(^6\) The equations necessary to fit a curve of the form of the first of these two expressions to a scatter of points by the method of least squares were developed. The nature of the resulting equations was such that, even for a few points, their solution would be tedious and time consuming.

\(^7\) M. Ezekiel, Methods of Correlation Analysis, John Wiley and Sons, New York, 1930.
Quality and Theory of Traffic Flow

**FIGURE V-2**

Speed-Density Plot — Merritt Parkway

\[ S = 53.2 e^{-\frac{D}{67}} \]

\[ r^2 = 0.90 \]
SPEED, VOLUME AND DENSITY RELATIONSHIPS

In Figure V-2 the data of Figure V-1 are plotted on semi-logarithmic paper, and it will be observed that a straight line gives a true indication of the trend of the data. The constants, obtained by a least-squares fit, are \( a = 53.2 \) and \( b = 67 \). For the log-linear plot \( r^2 = 0.90 \).

The data on which the curve of Figure V-2 is based has a density range of from approximately 20 v.p.m. to 120 v.p.m. For low densities it is more reasonable than Greenberg's equation in that it does give a finite speed as the density approaches zero, but it does not satisfy Guerin's requirement that its slope be zero at low density.

Two questions arise: First, does observation support the requirement of zero slope; and second, if so, is the departure of the model from it of practical significance? If the slope of the curve were in fact zero at low density, this would indicate that as the density (and hence the volume) approached zero, all desired overtakings could be accomplished. Such would be an expected result for a road on which there is no physical restriction to overtaking. Surprisingly, under these conditions, there are indications that as volumes approach zero only about ninety per cent of desired overtakings are performed.\(^1\) In cases where there is restriction preventing overtaking (as at the site where the data now being analyzed were collected), it could happen that, even under very light traffic, drivers could be influenced in the selection of their speeds by other vehicles on the road. The next set of data to be analyzed will support the reasonableness of the suggested model at low densities.

The model, however, breaks down at high densities because it is a curve asymptotic to the abscissa, although this is possibly not serious because it does not fail until densities well above the density corresponding to the possible capacity are reached. Figure V-3 gives speed-density and speed-volume plots with the necessary modifications for high densities shown. Actually, in practice, a roadway generally would not be subjected to densities so high that the exponential relationship would break down. On the infrequent occasions when it does, the straight-line modification should be satisfactory.

Alternatively, an expression of the form

\[ S = a e^{-\frac{D}{b - c}} \]

where \( c \) is a constant would satisfy the upper boundary condition.

---

\(^1\) O. K. Normann, "Results of Highway Capacity Studies," Public Roads, Vol. 23, No. 4, June 1942, p. 70.
QUALITY AND THEORY OF TRAFFIC FLOW

FIGURE V-3
S-V AND S-D CURVES — MERRITT PARKWAY

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SPEED, VOLUME AND DENSITY RELATIONSHIPS

but this is probably an unnecessary refinement, particularly as it complicates mathematical analysis.

Also shown in Figure V-3, for purposes of comparison, is a curve taken from the Highway Capacity Manual showing the maximum capacity of a traffic lane based on average spacings between pairs of vehicles traveling at the same speed. It will be noted that this curve and the lower arm of the speed-volume curve, derived from the exponential speed-density expression, are comparable.

For volumes and densities up to, and well beyond, the practical capacity, the speed-volume and speed-density curves in Figure V-3 closely approximate straight lines and, for all practical purposes, could be taken as such. The speed-volume curve deviates considerably from a straight line as the possible capacity is reached, but the speed-density curve is still reasonably close to a straight line up to the possible capacity; although beyond this limit, errors by use of a straight line would tend to become large.  

Probability Curves

The one-minute groups of data were next combined into the following density classification:

25 v.p.m. to 35 v.p.m., taken as 30 v.p.m.
35 v.p.m. to 45 v.p.m., taken as 40 v.p.m.
45 v.p.m. to 55 v.p.m., taken as 50 v.p.m.
75 v.p.m. to 85 v.p.m., taken as 80 v.p.m.
85 v.p.m. to 95 v.p.m., taken as 90 v.p.m.
95 v.p.m. to 105 v.p.m., taken as 100 v.p.m.
105 v.p.m. to 115 v.p.m., taken as 110 v.p.m.
115 v.p.m. to 125 v.p.m., taken as 120 v.p.m.

For each density grouping, cumulative frequency distributions of travel times were determined. These are plotted on arithmetic probability paper in Figure V-4. Particularly at the

---


10 Both Huber and Olcott fitted straight lines to data with densities in excess of the density at the possible capacity, with high coefficients of correlation. Huber's data covered the density range of about 20 v.p.m. to 140 v.p.m., but with no points between densities of about 50 to 80 v.p.m., while Olcott's data extended from 30 to 90 v.p.m. Figure V-3 indicates that for densities within these ranges, a straight line would closely approximate the curve. Extrapolation of their straight lines, however, leads to traffic jam densities that are too low.
QUALITY AND THEORY OF TRAFFIC FLOW

TRAVEL TIME DISTRIBUTIONS — MERRITT PARKWAY

FIGURE V-4

TRAVEL TIME — Seconds Per Mile

0.1 0.15 0.2 0.4 0.5 0.6 0.7 0.8 0.9 0.95 0.99 0.999

Percentage of Travel Times at or Greater Than Time Shown

450 400 350 300 250 200 150 100 50
higher densities, the trend is toward straight lines, indicating that travel times could be normally distributed. Accordingly, it was hypothesized that travel times were normally distributed with a mean and standard deviation equal to those of the measured distributions, and the hypothesis checked by the $\chi^2$ test.

The results of the $\chi^2$ tests are summarized in Table V-I.

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</tbody>
</table>

For densities of 90, 100 and 110 vehicles per mile there were no significant differences at the 0.05 level. At a density of 80 vehicles per mile, differences, while significant at the 0.05 level, were not significant at the 0.01 level. However, for the other densities, the differences were significant, and it would seem that these particular distributions could not reasonably be considered as normally distributed.

As the density decreases, drivers are less influenced in the selection of their speeds by other vehicles on the road. Thus, under "free speed" conditions, it could be expected that speeds, rather than travel times, would be normally distributed. The cumulative frequency distributions of travel times for densities of 50 vehicles per mile and less were then converted to cumulative frequency distributions of speeds. The resulting distributions were checked for normality, with the results shown in Table V-II.
QUALITY AND THEORY OF TRAFFIC FLOW

TABLE V-II
SPEED DISTRIBUTIONS — RESULTS OF CHI² TESTS

<table>
<thead>
<tr>
<th>Density v.p.m.</th>
<th>Calculated chi² value</th>
<th>chi² for rejection at 0.05 level</th>
<th>chi² for rejection at 0.01 level</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>7.9</td>
<td>7.8</td>
<td>11.3</td>
</tr>
<tr>
<td>40</td>
<td>5.7</td>
<td>3.8</td>
<td>6.6</td>
</tr>
<tr>
<td>50</td>
<td>3.7</td>
<td>9.5</td>
<td>13.3</td>
</tr>
</tbody>
</table>

Of these, the distribution for 40 vehicles per mile was significantly different from normal at the 0.05 level, but not at the 0.01 level, while the other two showed no significant difference at the 0.05 level.

The speed distribution for a density of 120 vehicles per mile was not checked because, when plotted on arithmetic probability paper, it departed considerably from a straight line and, hence, from normality.

As a result, it was concluded that at the lower densities the cumulative frequency distribution of speeds could, for the present purposes at least, be taken as normal. As density increases, the distribution of speeds becomes skewed because of the increasing effect on one another of vehicles in the traffic stream until, beyond a certain density, the departure of the distribution of speeds from normality is of practical significance. It turns out, however, that when this stage is reached, the distribution of travel times is close to normal. For the data now being examined, the distribution of travel times for densities of 80 to 110 vehicles per mile is practically normal.

These results are consistent with the findings of Berry¹¹ who suggested that, while for nonpeak conditions (at lower densities) travel-time data should usually be converted to speeds for purposes of analysis, it was preferable to work with travel times for certain peak conditions.

When the density increases to 120 vehicles per mile, the proportion of increased travel times, due to vehicular interaction, is such that travel times no longer can be considered as normally distributed.


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In Figure V-5 travel time distributions are plotted on the assumption that speeds are normally distributed for densities of up to, and including, 50 vehicles per mile, and that travel times are normal distributions for densities of 80 to 110 vehicles per mile. The curve for a density of 120 vehicles per mile is as observed. The curves have been drawn only over a percentage range of 5 to 95 per cent. Outside this range, the few observed points could deviate considerably from normality even though, as a whole, the distribution is close to normal.

It may be reasoned that under traffic-jam conditions (i.e., when vehicles are stopped bumper-to-bumper), a maximum density of about 220 to 240 vehicles per mile will be reached. At this point, since the speed is zero, travel times will be infinite. Bearing in mind this upper boundary condition, it is probable that the family of curves of travel time distributions could be extended as has been done in Figure V-5. It must be stressed that there is no factual basis for the additional curves -- they are supposition only -- but, nevertheless, are probably fairly close to what actually happens.

Figure V-6 shows a plot of the travel time-density curve derived from the speed-density curve of Figure V-2 and also a plot of the 50 percentile (or median) values read from Figure V-5. For truly normal curves the mean and median should coincide. However, the curves indicate that for densities of up to 110 vehicles per mile, the mean is greater than the median, although by only a slight amount. This discrepancy could possibly be due to the fact that extreme travel times in the data would assume different importance in the methods used to produce the curves of Figures V-2 and V-5. For instance, the curve of Figure V-2 was the result of a least-squares fit to every point throughout the full range of data while in Figure V-5 each of the density curves was arrived at by "fitting" normal curves to the data only within their range, and independently of each other.

Above a density of 110 vehicles per mile, the discrepancy is much greater with the median well below the mean. This can be explained. As density increases, the proportion of high travel times (or low speeds) increases due to vehicle interference, and the mean is more sensitive than the median to the alteration of these extreme values.

Figure V-6 indicates that for densities of up to 100 vehicles per mile, the 50 percentiles may be represented by a straight line. For higher densities, there is a marked deviation of the plot from a straight line. Other percentile values (10, 30, 70 and 90) were each plotted on semi-logarithmic scale for densities
FIGURE V-5
TRAVEL TIME DISTRIBUTIONS — MERRITT PARKWAY
FIGURE V-6
TRAVEL TIME-DENSITY CURVES — MERRITT PARKWAY

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of from 30 to 100 vehicles per mile. For each percentile the trend was linear. Curves were fitted to these points by least squares with the following results:

\[
\begin{align*}
    T_{10} &= \frac{98}{68.5} \frac{D}{e} \\
    T_{30} &= \frac{78}{64.3} \frac{D}{e} \\
    T_{50} &= \frac{71}{65.9} \frac{D}{e} \\
    T_{70} &= \frac{65}{67.6} \frac{D}{e} \\
    T_{90} &= \frac{64}{80.0} \frac{D}{e}
\end{align*}
\]

where \( T_{10} \) is the ten-percentile travel time, and so on.

It will be noted that the constant in the exponent of \( e \) is close to the value of \( b \) in Figure V-2; i.e., 67, the greatest difference being in the case of the ninety percentile curve. This is due largely to the 90 percentile value taken from the 30 v.p.m. curve of Figure V-5. For the other four curves, the mean of the constant is 66.6. As each constant differed only slightly from the mean, its value was taken as 66.6 (or practically 67) for all cases.

These equations are plotted in Figure V-7. The resulting family of curves indicates the probability of different travel times for varying densities. Also shown in Figure V-7 are speed-density probability curves, produced directly from the travel time-density curves. In Figure V-8, similar sets of curves have been drawn in terms of volume rather than density.

The volume curves indicate that the possible capacity of the facility is 1,320 vehicles per hour. Here an interesting point is the variation in speed and, consequently, travel time at the possible capacity. It has been suggested that "at the possible capacity all traffic must move at approximately the same speed, and the average difference in speeds between successive vehicles will approach or become zero."\(^\text{12}\) Also, the possible capacity of different facilities could be found by fitting a straight line to plots of mean difference in speed between successive vehicles and volume, and extrapolating to zero speed difference.\(^\text{13}\)

\(^{12}\) Committee on Highway Capacity, \textit{op. cit.}, p. 30.

\(^{13}\) \textit{Ibid.}, p. 31.
SPEED, VOLUME AND DENSITY RELATIONSHIPS

Curves show the probability of speeds greater than given values.

FIGURE V.7
S-D AND T-D PROBABILITY CURVES — MERRITT PARKWAY

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Curves show the probability of speeds greater than given values.

Curves show the probability of travel times equal to or greater than given values.

FIGURE V-8
S-V AND T-V PROBABILITY CURVES — MERRITT PARKWAY

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For the facility now being examined, this method of arriving at the possible capacity would not be valid. For a facility of long length, with ideal conditions, it could be that the mean difference in speeds between successive vehicles is close to zero at the possible capacity. However, in the case under consideration, it is reasonable that it should not be so. The section is a short bottleneck, with relatively free flowing conditions on either side of it. Therefore, traffic would arrive at the critical section approximately in a random manner with differing speeds and, at times, with appreciable differences in speed between successive vehicles. Thus, while overtaking is not possible over the section, there could be (and, in fact, was) a range of vehicle speeds. (At the possible capacity, the average headway was 2.73 seconds.) Had the bottleneck been a long section, a stage could soon be reached when all vehicles would form a long queue, at minimum headway and all at the same speed, with a resulting speed difference between successive vehicles close to zero. Should the volume of traffic arriving at a short bottleneck exceed the possible capacity of the bottleneck, a queue would rapidly develop back along the highway in advance of the restriction. It could turn out that the mean difference in speeds between successive vehicles might be less some distance in advance of, rather than at, the bottleneck itself.

Mean differences in speed between successive vehicles are not necessarily a reliable criterion on which to base the determination of possible capacity. However, should the exponential speed-density relationship be acceptable, it would readily lead to possible capacity computations. This will be discussed later in Chapter V-III.

PRINCES HIGHWAY

The data for this location were examined in a fashion generally similar to that just described, and much of the preceding discussion will apply to this section. Consequently, significant results will be given, but discussion will be kept to a minimum.

Speed-Density Relationship

The data were grouped generally into ten-minute intervals, with the exception of some of the higher volumes where groupings were taken over five-minute periods, and then plotted on a semi-logarithmic paper. As shown in Figure V-9, the trend seemed to be linear, and so a curve of the form

\[ S = a e^{-\frac{D}{b}} \]
QUALITY AND THEORY OF TRAFFIC FLOW

FIGURE V-9
SPEED-DENSITY PLOT — PRINCES HIGHWAY
was fitted by least squares with resulting constants of \( a = 51.4 \)
and \( b = 110 \), and with an \( r^2 = 0.86 \) for the log-linear plot.

Here the range of density is from approximately 4 to 60 vehicles per mile. The lowest densities are less than those recorded on the Merritt Parkway. The model still seems to hold reasonably well at these reduced densities suggesting that, even if the model does break down at extremely low density, its errors would not be serious. (At this site, a density of five vehicles per mile corresponds to a volume of approximately 250 vehicles per hour.)

The curve of Figure V-9 is replotted in Figure V-10. Also shown is the corresponding speed-volume curve, together with a curve\(^{14}\) showing the maximum capacity of a traffic lane based on average spacings between pairs of vehicles traveling at the same speed. The agreement between this latter curve and the lower arm of the speed volume curve is not as good as was obtained for the Merritt Parkway (Figure V-3). Unfortunately, at this site the density did not exceed that density corresponding to the possible capacity, so there is no means of checking the lower arm of the speed volume curve against actual observation. The extrapolated curve gives a reasonable value for the possible capacity -- 2,080 vehicles per hour -- although the speed of approximately 19 miles per hour at this volume seems too low. Thus, while the exponential speed-density curve fits the data well, it would appear that extrapolation of it to higher densities could give low speeds at points beyond the possible capacity.

Probability Curves

For the Merritt Parkway data it was shown that, for densities of 50 vehicles per mile and less, speed distributions were more closely normal than were travel-time distributions. Accordingly, for this data cumulative frequency distribution curves of speeds were drawn (Figure V-11) and checked for normality, the results being summarized in Table V-III.

The results indicate that, with the exception of the 30 vehicles per mile curve, the departure of each of the distributions from normality is insignificant at the 0.50 level. For the 30 vehicles per mile curve, it was found that the high \( \chi^2 \) value was due largely to the presence of a few high-speed vehicles on the road. When the actual and theoretical curves were compared, neglecting those portions of them above their 95 percentiles, \( \chi^2 \) reduced to 14.4. It was concluded that, for present purposes, the cumulative frequency distributions of speeds could be taken

\(^{14}\) Ibid., p. 28.
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Derived from S-D Curve below. Minimum speed at which volume may be attained. (after Normann)

FIGURE V.10
S-V and S-D CURVES — PRINCES HIGHWAY
SPEED, VOLUME AND DENSITY RELATIONSHIPS

TABLE V-III
SPEED DISTRIBUTIONS — RESULTS OF CHI\(^2\) TESTS

<table>
<thead>
<tr>
<th>Density v.p.m.</th>
<th>Calculated (\chi^2) value</th>
<th>(\chi^2) for rejection at 0.05 level</th>
<th>(\chi^2) for rejection at 0.01 level</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>13.6</td>
<td>18.3</td>
<td>23.2</td>
</tr>
<tr>
<td>10</td>
<td>11.2</td>
<td>16.9</td>
<td>21.7</td>
</tr>
<tr>
<td>20</td>
<td>5.7</td>
<td>16.9</td>
<td>21.7</td>
</tr>
<tr>
<td>30</td>
<td>41.9</td>
<td>18.3</td>
<td>23.2</td>
</tr>
<tr>
<td>55</td>
<td>9.9</td>
<td>9.5</td>
<td>13.3</td>
</tr>
</tbody>
</table>

as normal curves, without serious error.

Assuming the distributions to be in fact normal, the 10, 30, 50, 70 and 90 percentile speeds for each density classification were found, and for each percentile value, points of travel time versus density were plotted. As with the Merritt Parkway data, the trend in each case appeared linear. Least-squares fits to the sets of points gave the following results:

\[
T_{10} = \frac{D}{115}
\]

\[
T_{30} = \frac{D}{113}
\]

\[
T_{50} = \frac{D}{110}
\]

\[
T_{70} = \frac{D}{109}
\]

\[
T_{90} = \frac{D}{111}
\]

where \(T_{10}\) is the ten percentile travel time, and so on.

These curves are plotted in Figure V-12. The family of curves indicates the probability of different travel times for vary-
FIGURE V-11
SPEED DISTRIBUTIONS — PRINCES HIGHWAY
ing densities. Speed-density probability curves, derived directly from the travel-time curves, are also shown in Figure V-12. In Figure V-13, similar sets of curves are shown in terms of volume rather than density.

The curves of Figures V-12 and V-13 each have the same general shape as those of Figures V-7 and V-8. For a given density (or volume), speeds are higher (travel times lower), and the range of speeds (or travel times) between the 10 and 90 percent limits is less in the latter set of curves.

Extrapolation of the curves suggests that the possible capacity is 2,080 vehicles per hour. At this level, there is a range of speeds; this range is not as great as the previous example.

It has already been explained how Figures V-7 and V-8 were produced from the information contained in Figure V-5, and Figures V-12 and V-13 from that in Figure V-11. Figures V-5 and V-11, being cumulative frequency distribution curves, are affected by each individual speed; i.e., their shape, or slope is dependent on the scatter of the individual vehicle speeds. On the other hand, the points plotted in Figures V-2 and V-9 represent average values for definite time periods; i.e., they are means of samples of the universe. Consequently, their scatter is less than had individual values been plotted. If probability curves, based on these sample means, were produced, they would have less spread than the curves of Figures V-7, V-8, V-12 and V-13. Also, as the sample time period is increased, probability curves based on mean values for the period would be confined to a narrower band.
Curves show the probability of speeds greater than given values.

Curves show the probability of travel times equal to or greater than given values.

FIGURE V-12
S.D and T.D PROBABILITY CURVES — MERRITT PARKWAY
SPEED, VOLUME AND DENSITY RELATIONSHIPS

Curves show the probability of speeds greater than given values.

Volume - Vehicles Per Hour

Curves show the probability of travel times equal to or greater than given values.

Volume - Vehicles Per Hour

FIGURE V-13
S-V AND T-V PROBABILITY CURVES — PRINCES HIGHWAY

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Chapter V - III
FURTHER SPEED RELATIONSHIPS

EXPONENTIAL S-D CURVE

In Chapter V-II it was shown that curves of the form

$$ s = a e^{-bD} $$

fit the available data for both the Merritt Parkway and the Princes Highway with high coefficients of correlation. For the Merritt Parkway data, it was indicated that if the speed-density curve were extrapolated, it broke down as the density approached traffic-jam conditions. This deficiency did not seem serious, however, as a roadway generally would not be subjected to such high densities. On the few occasions that it was, a straight line modification could be used (Figure V-3).

For the Princes Highway where the density range of available data was not as large, the speed-volume curve, derived from the extrapolated exponential speed-density curve, gave a reasonable possible capacity. However, the speed at the possible capacity, and for points on the lower arm of the speed-volume curve in its vicinity, seemed low when compared with a curve showing the minimum speed at which any given volume may be attained (see Figure V-10).

In order to check further on the reasonableness of a proposed model, some data used by Guerin\textsuperscript{1} and collected by him at a site in Temple Street, New Haven, was plotted on semi-logarithmic paper (Figure V-14). A straight line was drawn through, but not statistically fitted to, the resulting points. The straight line gives a good indication of the trend of the points. It gives a possible capacity of 670 vehicles per hour for the site, at a speed of nine miles per hour. The data were for a single-lane section close to traffic lights, and hence these figures appear reasonable.

May and Wagner\textsuperscript{2} have produced speed-volume curves for a site on the Ford Expressway in Detroit -- a six-lane divided facility. These have been replotted in terms of speed and density on semi-logarithmic paper in Figure V-15, and it is evident that their trend is nonlinear.

\textsuperscript{1} N. S. Guerin, "Travel Time Relationships," Bureau of Highway Traffic, Yale University, Student Thesis, 1958, p. 32.


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FIGURE V-14
SPEED-DENSITY PLOT — TEMPLE STREET
QUALITY AND THEORY OF TRAFFIC FLOW

Also plotted in Figure V-15 are curves representing conditions on the Arroyo Seco Parkway near Los Angeles. These were deduced from speed-volume curves presented by Forbes. These are approximately linear, but indicate that the possible capacity is of the order of 5,000 vehicles per hour per lane, which is unrealistic.

It must therefore be concluded that the suggested exponential speed-density relation is not applicable to high-type facilities, although it may be used with advantage on lower-class facilities. Some of the advantages of the speed-density relationship are as follows:

(a) It plots as a straight line on semi-logarithmic paper. Hence, theoretically, only two points are necessary to establish it, although the larger the number of points, the greater would be the accuracy attained.

(b) It may be fitted to a scatter of points by the method of least squares with relative ease.

(c) It may readily be transformed to give expressions relating any of the variables -- speed S, time T, density D or volume V. These expressions are as follows:

\[ S = a e^{-\frac{D}{b}} \]
\[ T = a_1 e^{\frac{D}{b}} \quad \text{where } a_1 = \frac{1}{a} \]
\[ V = \frac{Sb}{\ln \frac{a}{s}} \]
\[ V = \frac{b}{T} \ln (aT) \]
\[ V = aD e^{\frac{D}{b}} \]

(a and b are constants, e is the exponential constant.)

(d) The properties of the curve are such that, once the equation to it is established, the constants lead to important flow properties as follows:

possible capacity = \( \frac{ab}{e} \)

speed at possible capacity = \( \frac{a}{e} \)

density at possible capacity = \( b \)

FIGURE V - 15
SPEED-DENSITY PLOTS — FORD EXPRESSWAY AND ARROYO SECO PARKWAY

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"free" speed = a (providing it can be shown that the model holds for extremely low densities)

PROPOSED GENERAL S-V RELATIONSHIP

In this section a general speed-volume relationship, which it is thought could have general application to all types of highways, will be put forward. It will then be examined in the light of available data, and in terms of the various existing models previously discussed.

Suggested Relationship

A suggested general speed-volume relationship is illustrated in Figure V-16. It envisages traffic flow being subdivided into three separate zones -- a zone of normal flow, a zone of unstable flow, and a zone of forced flow. Each of these will be discussed in turn.

Zone of Normal Flow

Here it is assumed that there is a linear relation between speed and volume.

Possibly the most extensive studies conducted to determine the effect of volume on speed are those by Normann. He concluded that there was a straight-line relation between speed and volume provided a certain critical density was not exceeded. It has already been shown that for certain lower-type facilities, an exponential curve will well fit speed-density data over a wide density range. This would result in a curvilinear speed-volume curve, but through the zone of normal flow a straight line would fit the data equally well.

It is a characteristic of speed-volume data that, when plotted, points do not fall along a definite line. Rather, they form a scatter, and speed-volume probability curves have already been produced (Figures V-8 and V-13). To completely describe the zone of normal flow, probability lines may be drawn as in Figure V-16. In this figure, 80 per cent of observed points would lie between the 10 per cent and 90 per cent limits.

The zone of normal flow will alter for different facilities. The intercepts on the speed axis, the spacing of the probability lines and their slope will (or may) all vary.

Zone of Unstable Flow

As the volume increases, a point will be reached when the flow will break down. At this point there will be a marked reduc-
Curves show the probability of speeds greater than given values.

Note: All zones have been shown bounded by the 10 and 90 percentiles. In fact, they would extend beyond these limits.

FIGURE V-15
GENERAL SPEED-VOLUME DIAGRAM
QUALITY AND THEORY OF TRAFFIC FLOW

tion in speed, with consequent increase in density. The precise point of breakdown is indefinite, although it is postulated that it can be specified in terms of probabilities, along the lines indicated in Figure V-16. From data at present available, it seems that when the density reaches approximately 60 vehicles per mile, the flow will break down about 50 per cent of the time.

There would be no definite relation between speed and volume in the zone of unstable flow. Rather it would be a transition between the zones of normal and forced flow. Depending on the degree of breakdown, traffic flow may drop into the forced-flow zone and remain there. It may oscillate between the normal and forced flow zones, or it may momentarily drop to the forced flow zone and return to a stable condition in the normal flow zone.

It may be reasoned that an unstable zone of flow must exist. At extremely low volumes (and with low density) vehicles travel at a speed well above zero -- generally at, or close to, their free speed, excluding exceptional circumstances. Under traffic-jam conditions, when all vehicles are stopped, the volume being cleared is zero. For volumes between zero and the possible capacity, speeds are somewhere between the two extreme limits; i.e., between free speed and zero. The relation between speed and volume may be expressed as

\[ s = f(V) \]

where \( f(V) \) is some function of \( V \).

White has concluded that at the point of maximum volume the speed-volume curve should be continuous and smooth, and Guerin has accepted his conclusion.\(^4\)

The rate of change of speed with respect to change of volume will be given by the first differential of the expression above,

\[ \text{i.e., } \frac{ds}{dV} = f'(V) \]

If the \( S-V \) curve is continuous, this latter expression will be infinite at the point of maximum volume (the possible capacity), and will have finite values on either side of the maximum. Hence, it is impossible for flow to be maintained at the possible capacity. At points close to the possible capacity, since \( \frac{ds}{dV} \) is large, small volume surges will tend to move operating conditions to the possible capacity, thus giving rise to unstable conditions.

\(^4\) N. S. Guerin, op. cit., p. 39.
FIGURE V-17
GENERAL S-V DIAGRAM — COMPARISON WITH OBSERVATION
QUALITY AND THEORY OF TRAFFIC FLOW

Some of the curves produced by Palmer indicate clearly the presence of an unstable area in the vicinity of the possible capacity.¹

It is postulated that the zone of unstable flow is sensibly constant for all facilities, and that within it, flow cannot be maintained at a constant rate for any length of time. Rather it is a transition between the zones of normal and forced flow.

Zone of Forced Flow

This is the third zone. As with the zone of unstable flow, it too, is thought to remain substantially constant for all facilities. As with the other zones, it may be defined on a probability basis, and the 50 percentile line is assumed to coincide with Normann's line representing the maximum capacity of a traffic lane based on average spacings between pairs of vehicles traveling at the same speeds.

Comparison with Observation

The proposed general speed-volume relation will now be shown to be consistent with available observed data. Figure V-17 shows speed-volume data for sites on the Merritt Parkway (derived from Figure V-2), Temple Street (derived from Figure V-14) and the Ford Expressway.⁶ In each case the points represent average volumes and speeds for one-minute periods. For each of the facilities, the zone of normal flow would differ, and only the 50 per cent curves for these differing zones are shown. The figure indicates that the observed data could well fit the two constant zones; i.e., the unstable and forced flow zones.

Figure V-18 shows speed-volume plots for the Princes Highway (from Figure V-10), and for California highways, these latter curves being reproduced from papers by Forbes⁷ and Webb and Moskowitz.⁸ Whereas the curves in Figure V-17 are based on the average of observations for one-minute periods, the curves in Figure V-18 are generally based on five-minute groupings of data. In the case of the Princes Highway, however, some of the low volume points represent average conditions over a ten-minute period. Also shown are the postulated unstable and forced flow zones, these being drawn exactly as in Figure V-17.

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⁶ A. D. May and F. A. Wagner, op. cit.


FIGURE V-18
GENERAL S-V DIAGRAM — COMPARISON WITH OBSERVATION
QUALITY AND THEORY OF TRAFFIC FLOW

Before drawing conclusions from this diagram two factors should be considered.

Firstly, the various zones of the general speed-volume relation are expressed in terms of probabilities. Because of this, the length of the time periods used to classify the data are of importance. It has just been mentioned that the curves of Figure V-18 are generally based on five-minute groupings, while those of Figure V-17 are the result of one-minute groupings. As the length of this grouping period increases, the scatter of points within the normal flow and forced flow zones would become less, giving a narrower band between given probability limits. At the same time, the width of the zone of unstable flow, between given probability limits, would tend to increase, because the longer a high volume is maintained at a given mean value, the greater is the probability of a larger deviation above the mean. Hence, the greater the probability of a breakdown of flow. Figure V-19 shows the effect on the various zones of the speed-volume diagram of varying the grouping time interval. Figure V-20 is a replot of Figure V-18, but with the zones of unstable and forced flow modified to take account of this variation.

The second point concerns the shape of the plotted curves which deviate, with varying degrees, from a straight line. These curves could be expressed as

\[ V = -aS^2 + Sb - c, \]

and it could be that the probability curves representing the normal flow zone should be of this form. In his paper, Forbes shows the points on which his curves have been based, and while there is a slight curvilinear trend, straight lines could be fitted with almost as high a degree of precision. Webb and Moskowitz, on the other hand, do not show the points from which their curves have been derived. Their curves were constructed by reading values from two other graphs, each of which were fitted, by inspection, to plotted points. As they, themselves, comment, "considerable freedom was permitted in the fitting, resulting in sinuous curves which are close to group-averaged plotted points." It is not possible, therefore, to ascertain the magnitude of error that would be introduced by fitting straight lines, rather than parabolic arcs, to their points.

The departure of the speed-volume curve from a straight line is explainable. If a high volume is maintained at a given

11 Loc. cit.
FIGURE V-19
EFFECT ON S-V DIAGRAM OF INCREASING TIME INTERVALS
QUALITY AND THEORY OF TRAFFIC FLOW

**Arroyo Seco Parkway,**
**Pasadena Freeway**
**Princes Highway**

**Volume - Vehicles per Hour**

**Zone of unstable flow**

**Zone of forced flow**

**Hollywood Freeway**

**Volume - Vehicles Per Hour**

**Zone of unstable flow**

**Zone of forced flow**

FIGURE V-20
GENERAL S-V DIAGRAM — MODIFICATION OF FIGURE V-18

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FIGURE V-21
SPEED-VOLUME THEORIES - MERRITT PARKWAY

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mean value for, say, five minutes, then because of flow variations in the period there could be occasional short "localized" flow breakdowns that would tend to lower the mean speed for the period, but which would not cause complete breakdown.

This curvilinearity of the speed-volume line at high volume is in conformity with the concept of the unstable zone with an increasing probability of flow failure as the volume increases, and it seems that marked deviation of the curve from a straight line would indicate that the flow was within the unstable zone.

Actually, it could happen that nonlinearity of the speed-volume curve in this region is of little practical importance. It is envisaged that the main use of data within this range of higher volumes would be in connection with traffic monitoring on expressways, and for this it would be necessary to work at a fairly low probability level of flow collapse -- possibly of the order of ten per cent or so. This being so, the linear relation would be adequate.

Comparison with Other Models

Several of the models of traffic flow that have been proposed from time to time have been discussed, and some of their apparent deficiencies mentioned. Although they differ from one another, all can, for the most part, be reconciled with the proposed generalized speed-volume diagram.

Figure V-21 shows a speed-volume diagram for the Merritt Parkway together with speed-volume plots\(^{12}\) according to the theories of Greenshields, Normann and Greenberg.

General comments on Figure V-21 are as follows:

1. The observed points have a large scatter, so that curves, supposedly indicating average conditions, do not have much real meaning, whereas zones defined in terms of probabilities seem more realistic.

2. Observed points falling within the zone of forced flow have no part in fixing the zone of normal flow in the speed-volume diagram. This, too, is true for Normann's curve, but with the other two curves shown, every point has equal weight in positioning the complete curve. It is thought that the points falling within the unstable flow zone should be disregarded when fixing the normal flow zone. Some of these points would represent normal

\(^{12}\) These speed-volume plots have been derived from the speed-density curves of Figure V-1. It will be recalled that the curves of Figure V-1 were not statistically fitted.
FIGURE V - 22
GENERAL S-D DIAGRAM — MERRITT PARKWAY

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conditions, but others could have been affected by either complete or localized failure and, therefore, it seems best to neglect all of them when fixing the normal flow zone.

(3) The Greenshields' model is generally in accordance with the speed-volume diagram, the main difference being in the vicinity of the possible capacity. It is now suggested that at this point flow is unstable, as compared with Greenshields' continuous curve.

(4) Also, in the case of Normann's curve, the major difference is in the vicinity of the possible capacity. In one of his later papers, Normann does provide for the possibility of an unstable condition at the possible capacity.\(^\text{13}\) Within the forced flow zone, both are in close agreement. Figure V-21 shows some discrepancy between the normal flow zone and Normann's curve. This is due largely to different means of producing them. The normal flow zone was located by using only those points not within the other two zones, whereas Normann's curve was based on all points except those in the forced flow zone.

(5) Greenberg's curve, like that of Greenshields, is continuous at the possible capacity. It was mentioned earlier that at low volumes (and densities) Greenberg's equation gives unrealistic results because, as the volume approaches zero, the density approaches infinity.

S-D and V-D Diagrams

The generalized speed-volume diagram of Figure V-21 has been reproduced in two different forms in Figures V-22 and V-23. Figure V-22 shows the diagram in terms of speed and density, together with the corresponding Greenshields and Greenberg curves. It indicates that Greenshields' curve gives results similar to those of the more general diagram for densities of up to about 120 vehicles per mile. For higher densities his model seems to break down, in that it gives traffic-jam densities that are too low. Greenberg's equation, on the other hand, fits well in the higher density range, but for low densities it gives speeds which are much too high. Figure V-23 is a volume-density diagram. Both Figures V-22 and V-23 indicate clearly a definite separation between the normal and forced flow zones; i.e., the presence of a transition or unstable zone.

It is stressed that the probability limits shown in Figure V-21 and 22 are not the same as those of Figures V-7 and V-8. Figures

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V-7 and V-8 were developed by considering each individual observation (vehicle) in the traffic stream, whereas the points plotted in Figures V-21 and V-22 are one-minute averages, and hence, in these later two figures, the spread between given probability limits should be less.

FIGURE V-23
GENERAL V-D DIAGRAM — MERRITT PARKWAY
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CONCLUSIONS

Families of curves, expressing relationships between travel time and density, speed and density, travel time and volume, and speed and volume in terms of probabilities are a practical, realistic method of summarizing traffic data.

For certain lower type facilities, there seems to be, for practical purposes, an exponential relationship between the space mean speed, $S$, of traffic and its density, $D$. It is of the form:

$$S = ae^{-\frac{D}{b}}$$

where $a$ and $b$ are constants.

However, this relationship does not hold for expressway type facilities.

A general speed-volume diagram has been put forward. It postulates that traffic flow can best be described in terms of three distinct zones -- a zone of normal flow, a zone of unstable flow, and a zone of forced flow -- each zone being specified in terms of probabilities.

The diagram is consistent with available observed data. It can be reconciled with the models of Greenshields, Normann and Greenberg, and at the same time, seems to overcome their apparent deficiencies.

The concept of three distinct flow zones, if developed further, could lead to a clearer understanding of the theory of traffic flow, and in this regard could be a valuable aid to those seeking a sound theoretical solution of the problem.