Empirical Relation between Stochastic Capacities and Capacities Obtained from the Speed-Flow Diagram

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Introduction

**Capacity** = Maximum possible throughput that can be expected under specific geometric, traffic, and control conditions
Introduction

Conventional Design Capacities (HCM, HBS):

• Capacity is treated as a constant value
• Derivation of design capacities for freeways is usually based on the analysis of speed-flow diagrams

Stochastic Capacities:

• Capacity is regarded as a random variable
• Capacity distribution function represents the traffic breakdown probability in dependence on the flow rate
• Design capacities could be defined as a specific percentile of the capacity distribution function
Objectives:

• Analyze the empirical relationship between conventional and stochastic capacities based on data samples from German freeways

• Derive breakdown probabilities that correspond to conventional design capacities given in traffic engineering guidelines
Conventional Capacity Analysis

Capacity Estimation in the Speed-Flow Diagram:

![Graph with speed on the y-axis and flow rate on the x-axis]

- speed (km/h)
- flow rate (veh/h)
Conventional Capacity Analysis

**Van Aerde’s Model:**

\[ d(v) = \frac{1}{c_1 + \frac{c_2}{v_0 - v} + c_3 \cdot v} \]

- Minimum desired headway between consecutive vehicles

- Speed-flow-density relationship is described by a continuous function

- Model parameters \( v_0, c_1, c_2 \) and \( c_3 \) can be calibrated by non-linear regression in the speed-density plane
Conventional Capacity Analysis

Aggregation of Different Traffic States within 1-Hour Intervals:
Conventional Capacity Analysis

Confidence Limit for the Capacity Estimate:

\[ c_{\text{model}} < q_{99\%} ? \]
Conventional Capacity Analysis

Empirical Capacity Estimates vs. HBS Design Capacities:

- 2 lanes, no speed limit
- 3 lanes, no speed limit
- 2 lanes, const. speed limit
- 2 lanes, variable speed limit
- 3 lanes, variable speed limit
- 2 lanes, uphill section
- 3 lanes, uphill section
Stochastic Capacity Analysis

Random Variability of Breakdown Volumes:

censored intervals: $c > q$

pre-breakdown intervals: $c = q$

congested intervals
Empirical Estimation of Capacity Distribution Functions:

- Non-parametric estimation: Product-Limit-Method
  \[
  F_C(q) = 1 - \prod_{i : q_i \leq q} \frac{k_i - 1}{k_i}; \quad i \in \{Z\}
  \]

- Parametric estimation: Maximum-Likelihood-Method
  \[
  L = \prod_{i=1}^{n} f_C(q_i)^{\delta_i} \cdot \left[1 - F_C(q_i)\right]^{1-\delta_i}
  \]
  Weibull function: \( F_C(q) = 1 - e^{-\left(\frac{q}{b}\right)^a} \)
Stochastic Capacity Analysis

Estimated Capacity Distribution Function (5-Minute Intervals):

- Max.-Likelihood estimation
- Product-Limit estimation

- Estimated Capacity Distribution Function (5-Minute Intervals): 4760 veh/h
Transformation between 5-Minute and 1-Hour Intervals:

\[ 1 - F_{C,60}(q_{60}) = \prod_{i=1}^{12} [1 - F_{C,5}(q_{5,i})] \]

\[ q_5 = N(q_{60}; \sigma_{q_5}) \text{-distributed} \]
Relation between Stochastic and Conventional Capacities

Breakdown Probability Corresponding to the Nominal Capacity:

![Graph showing relation between speed, flow rate, and capacity distribution function. The graph includes 1-hour speed-flow data, 1-hour van Aerde model, 5-minute Weibull distribution, and 1-hour Weibull distribution. It highlights points F_{c,60}(c_N) and F_{c,5}(c_N) corresponding to the nominal capacity c_N.]}
Empirical Results for 27 Freeway Cross Sections in Germany:

- Average 5-minute breakdown probability: $F_{c,5}(c_N) = 3\%$
- Average 1-hour breakdown probability: $F_{c,60}(c_N) = 40\%$
Relation between Stochastic and Conventional Capacities

Further Empirical Findings:

• On sections with variable speed limits, the capacity variance tends to be smaller than on sections without a speed limit.
• With temporary hard shoulder use, the average capacity of a 3-lane carriageway is increased by 20-25%.
• The road gradient does not influence the capacity variance.
Conclusions

• Conventional design capacities for basic freeway segments are usually derived by analyzing the speed-flow diagram

• Stochastic approaches deliver a capacity distribution function, which represents the probability of a traffic breakdown in dependance on the flow rate

• Defining a maximum acceptable breakdown probability could be an alternative way to derive freeway design capacities

• On average, the 1-hour freeway design capacities given in the German Manual HBS correspond to a 40 % breakdown probability during one hour