

**ADAPTING CAR TRAFFIC MODELS AND CONCEPTS TO BICYCLE  
TRAFFIC**

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## **ABSTRACT**

Over the past 50 years, a huge number of traffic models have been developed to capture and model the driver behaviour on streets, roads and freeways. Almost, all models were developed for motorized modes. Hence, as bicycle use in cities rapidly increases the need for more refined models arises in order to capture and model cyclist behaviour on bike facilities.

In order to capitalize on the years of traffic modeling development, it is legitimate to consider adapting these models to bicycle traffic and thus benefit from all the research efforts in the field of traffic theory. Since, the objective function for a driver, or a cyclist, is to control his vehicle's speed and direction while maintaining his desired speed and avoiding accidents; it is likely, that despite the difference in vehicle type, the driving logic and driver behaviour will be similar. Research has already begun in this area (1, 2), but compared to what has been done for motorized modes, we are still in the early stages of traffic science for this emerging mode.

This paper presents three aspects of traffic vehicle science adapted to bicycle traffic. These are car-following models (3, 4, 5, 6, 7, 8), the fundamental relationships of traffic flow (5, 9, 10, 11) and the Action Point model (12). The results obtained using car-following models were compared to empirical data collected with GPS devices installed on a pair of cyclists in a following situation with no opportunity to overtake. The fundamental relationships are examined using data collected by video at a fixed location on a bike facility. Finally, the paper presents a behavioural comparison between the bike-cyclist and the car-driver systems using the Action Point model.

The results indicate that car-following models, fundamental relationships and the Action Point model all have the potential to reproduce real world data for bicycle traffic. Also, the commercial simulation software, in this case, VISSIM (18), can be used with some calibration effort, to reproduce similar results to the observed data.

## **INTRODUCTION**

New realities have emerged with the growing of cycling as a transportation mode in its own right. Thus, cycling is no longer reserved to leisure and its utility now exceeds its recreational use. In addition, occasionally we begin to see on some bicycle facilities in Montréal (Québec, Canada) a phenomenon long reserved to motorized modes: congestion i.e. situations where platoons of cyclists are forced to follow in single lane given the density of cyclists and lack of overtaking opportunity. This situation can be well modeled by bike-following model.

The need to develop methods and tools to assist engineers and planners to analyze the performance of this mode is increasingly justified (1). Researchers (2, 3) have already begun but compared to what has been done for motorized mode, we are still in the early stages of the traffic science for this emerging mode.

## **DATA COLLECTION**

Two types of data were collected as part of this study in two locations in Montréal (Québec, Canada). Both locations were off- street, two-way exclusive bicycle facilities.

### **GPS data**

The first set of data was collected in a controlled experiment. To examine and model the behaviour of a pair of cyclists in a strictly following situation, the time-space trajectories of a pair of cyclists on bicycle facility, was recorded by a two GPS devices (Garmin Forerunner 305). Every second, the units would provide speed and position for a pair of cyclists.

Both devices were synchronized at starting point and secured to wrists of each cyclist, allowing the recording of data on a stretch of 1.7 km on bicycle facility of 1.50 m wide in Montréal. The raw data was then compiled and processed to build the time-space diagram. The time-space diagram is very rich in information, enabling one to calculate an important set of indicators that explain the behaviour of the cyclists.

Despite the precision of this GPS device (velocity accuracy  $< 0.05$  m/s), some data were cleaned from random errors, attributed essentially to satellite signal blockages. Less than 6% of raw data was smoothed to minimize the observed errors. The smoothed process consisted to calculate the new values from the average of six values data observed before and after the error (three on each side).

### **Video data**

A second set of data was recorded on a 1.45 m wide exclusive off-street bicycle facility located in Montréal, using a video collection unit. This recorded the times of passages of bicycles passing over a short section of 2 meters. A camera mounted on a 6 meters high pole was filming the flow of bicycles during the PM peak hour. From the 324 cyclists

recorded, 253 pairs of cyclists were identified. These pairs were within platoons of 2 to 15 cyclists. The data was extracted manually with the Windows Movie Maker utility from Microsoft that allows the processing with a precision of one thirtieth ( $1/30^{\text{th}}$ ) of a second. We could then obtain the individual speed, the spacing, the gap and the headway for each pair of cyclists. The dataset would then yield through mathematical transformations to flow rate, density and speed.

### Global data Analysis

The dataset collected provides a rich amount of information on the dynamics of the cyclist and his behaviour. Regarding of the basic parameters (speed, acceleration and spacing) Table 1 gives the general statistics compiled using data collected with the GPS device as well as the recorded video.

**TABLE 1 Statistics of Bicycles Dynamics**

Parameters	Minimum	Maximum	Average	85 <sup>th</sup> percentile	Standard Deviation	N
Speed (m/s) (from video)	2.00	8.00	5.67	6.42	0.99	324
Acceleration (m/s <sup>2</sup> ) (from GPS)	--	1.34	0.20	0.40	0.21	340
Deceleration (m/s <sup>2</sup> ) (from GPS)	-1.81	--	-0.20	-0.03	0.22	340
Spacing (m) (from video)	2.89	62.91	14.78	24.93	10.09	253
Spacing (m) (from GPS)	2.22	61.10	13.2	23.17	10.00	340

These values are quite similar to those found in the literature. For example, the average speed used by the Highway Capacity Manual 2010 (4) is 5.7 m/s. In general, the average speed reported in the literature (5, 6) varies between 4.7 m/s and 6.9 m/s. As for the acceleration, one of the rare studies on the subject in Denver, Colorado (6) reported an average acceleration of 0.24 m/s<sup>2</sup> and an average deceleration of -0.27 m/s<sup>2</sup> on an off-street facility.

### BIKE-FOLLOWING MODEL AND RESULTS

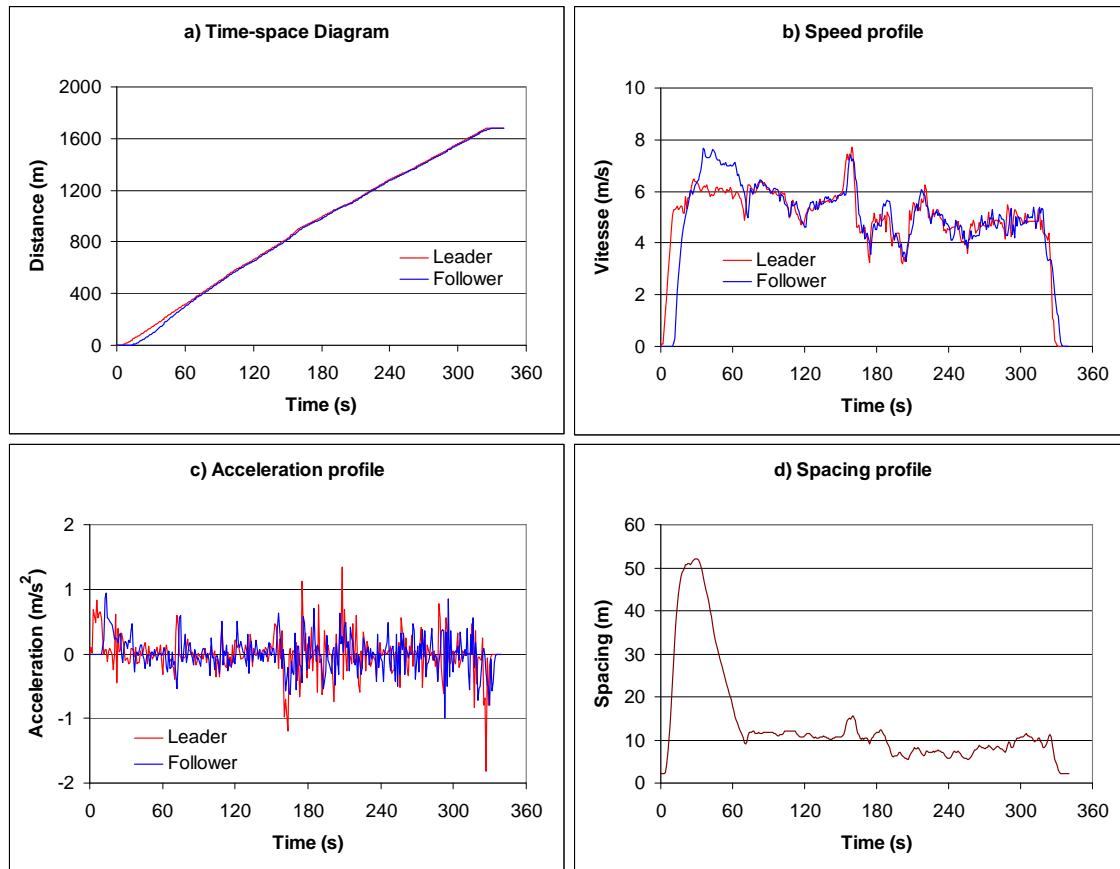
As bicycle traffic increases, a cyclist increasingly needs to evaluate and adjust his relative position to other cyclists, thus the importance of modeling the bike-following situation. A bike-following model is a single mathematical equation combining direct and indirect physical and operational characteristics of the bicycle (length, speed and acceleration), human factors (perception, decision and action) and the cyclist's immediate environment (heavy, moderate or free flow traffic). Overall, the microscopic model is formulated using the following logic:

$$Reaction_{(t+\Delta t)} = f(Sensibility_{(t)}, Stimulus_{(t)})$$

Over the past 50 years, close to a hundred car-following models have been developed (7). These models were originally intended for the understanding the traffic dynamic, more specifically the phenomenon of congestion. These models are also used in traffic safety analysis, and since the late 60's, they are imbedded in microscopic traffic simulators. Currently, microscopic simulation is extensively used for planning, training and research. As of the mid-90's, microscopic simulation has become an integral part of intelligent transportation systems.

### Microscopic Data Analysis

The data collected with the GPS device allowed reproducing the actual behaviour of both cyclists during the trip using one-second time steps. Figure 1 show the trajectory of the pair of cyclists, their speed and acceleration profiles as well as the spacing maintained between the two bicycles.

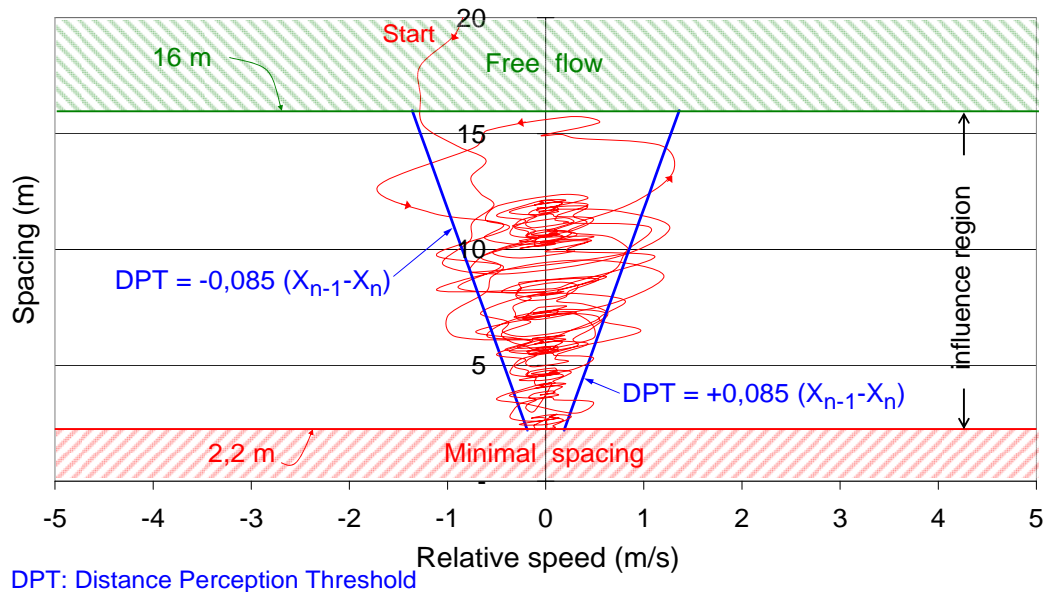


**FIGURE 1 Behaviour of a pair of cyclists in following situation**

The previous figure shows the correlation between the each cyclist's dynamic behaviour, especially when the spacing between them is reduced, which occurs around the sixtieth second. At this moment, the follower is forced to adjust his speed to the leading cyclist because the follower can not overtake the leader. The follower's set point

was to start 10 seconds after the start of the first cyclist, then he had to catch up and follow the leader without overtaking him.

Thus, there exists a given spacing between two cyclists beyond which the follower moves freely and below which he needs to adjust his speed to the leader to maintain a safe distance while trying to maintain his desired speed. This can be better illustrated using a phase plane of the first order, also known as Action Point Model, as shown in figure 2. This graph links the relative velocity (speed difference of the two cyclists) to the spacing between the two cyclists (8).



**FIGURE 2 Phase plane trajectory for a pair of cyclists in following situation**

From figure 2, it is found that the follower enters the leading cyclist's influence zone when their spacing is about 16 m. Kahn and Raksuntorn (6) found this interaction distance equal to 21 m. In a natural process, the follower tries to minimize the difference between his and the leader's speed while maintaining a safe and comfortable distance. The equilibrium is achieved through several adjustment cycles by the follower. This equilibrium point is not static but varies depending on the relative speed and spacing.

In order to adjust his speed, the follower must continually be aware of his position relatively to his equilibrium point. This information is provided through the Distance Perception Threshold (DPT). This is defined as the minimum level of change in spacing that can be perceived by the cyclist. At this threshold, the cyclist becomes aware that he is moving away from his equilibrium point. If he is negatively moving away from this point, he will increase his speed and if he is positively moving away, he will reduce his speed. The simplest equation, developed by Evans (9), was chosen and adapted to bicycle traffic.

The same figure shows the minimal distance the cyclist keeps at all times to avoid colliding with the leading cyclist, when overtaking is not possible. This minimal spacing

corresponds to the length of a bicycle (1.8 m) plus a minimal gap. According to field data collected with the GPS device, this minimal gap is about 0.42 m; therefore, resulting in a minimal spacing of 2.22 m. However, the video data indicated that some following cyclists had a lateral offset to the leader but just behind him in the same lane. So, there was no spacing between the two cyclists. In the most case, this situation was a prelude to the overtaking manoeuvre. These cases were not taken into consideration for this study.

### Bike-following models

Five models representing the bike-following situation were investigated. The choice is mainly based on the popularity of models and their theoretical concept. In addition, a sixth model proposed by the authors has also been the object of the analysis. Table 2 shows the models used to reproduce the behaviour of the following cyclist according to his own behaviour and the behaviour of the leading cyclist. The first four models are derived from existing car-following models, the fifth developed specifically for bikes and the last is proposed by the authors.

**TABLE 2 A List of Bike-Following Models Investigated**

Models	Formula
Chandler (10)	$a_n(t+T) = \lambda \left[ v_{n-1}(t) - v_n(t) \right]$
Gazis (11)	$a_n(t+T) = \alpha \frac{v_n(t)^m}{\left[ x_{n-1}(t) - x_n(t) \right]^l} \left[ v_{n-1}(t) - v_n(t) \right]$
Pipes (12)	$a_n(t+T) = CW \frac{v_{n-1}(t) - v_n(t)}{\left[ x_{n-1}(t) - x_n(t) \right]^2}$
Gipps (13)	$v_n(t+T) = \text{Min} \left\{ \begin{array}{l} v_n(t) + 2,5aT(1-v_n(t)/V)\sqrt{0,025 + v_n(t)/V} ; \\ bT + \left[ b^2T^2 - b \left\{ \begin{array}{l} 2[x_{n-1}(t) - x_n(t) - s] - v_n(t)T \\ - v_{n-1}(t)^2/b^* \end{array} \right\} \right]^{1/2} \end{array} \right\}$
Raksuntorn (6)	$v_n(t+\Delta t) = \alpha_1 v_n(t) + \alpha_2 \left[ x_{n-1}(t) - x_n(t) \right] + \alpha_3 \left[ v_{n-1}(t) - v_n(t) \right]$
Proposed Model (Manar)	$v_n(t+T) = \text{Min} \left\{ V_f ; \delta \left( x_{n-1}(t) - x_n(t) - S_j \right) \right\}$

The first three models simulate the acceleration and the other three simulate the speed. The model proposed by the authors incorporates two constraints. The first

constraint reflects a minimum spacing ( $S_j$ ) between two stopped cyclists. A second constraint limits the maximum speed of the cyclist, when the free flow conditions prevail. It is assumed that this speed corresponds to the 85<sup>th</sup> percentile of observed speeds.

For the calibration process, the values of the proposed models parameters were optimized using an iterative function minimizing the sum of squared deviations. This optimization allowed reproducing as faithfully as possible the four variables describing the behaviour of two cyclists in a following situation. These variables are speed, acceleration, position and the spacing between the two bicycles for every time step (1 sec).

In addition, for each variable, we calculated the regression between observed and theoretical values obtained by each model. The value of the slope of the regression and the multiple correlation coefficients ( $R^2$ ) were obtained. Also, the Fisher significance test was calculated to assess the similarity between the observed and theoretical values. Table 3 presents the optimized parameters and the statistical comparison between the observed and theoretical data.

**TABLE 3 Bike-Following Models Comparisons**

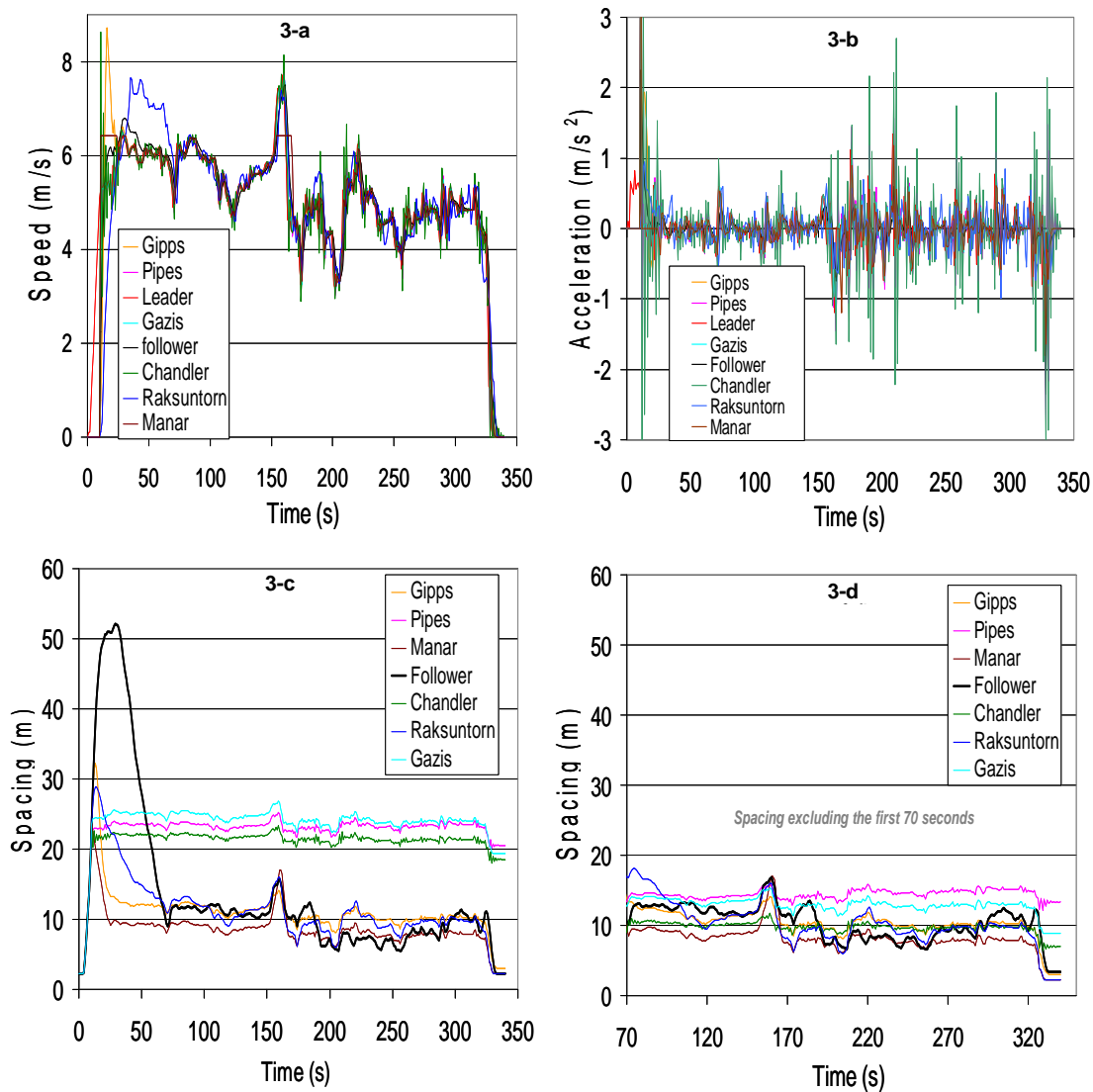
Models		Chandler	Gazis	Pipes	Gipps	Raksuntorn	Manar
Parameters							
Optimized Coefficients		$\lambda = 1.7$	$\alpha = 1.7$ $m = 0.01$ $l = 0.17$	CW = 680	$a = 2.0$ $T = 1.0$ $V = 10.0$ $s = 3.0$ $b = -2.0$ $b^* = -2.0$	$\alpha_1 = 0.96$ $\alpha_2 = 0.02$ $\alpha_3 = 0.35$	$V_f = 6.42$ $\delta = 0.85$ $S_j = 2.22$
Speed	Slope	0.999	1.008	1.006	0.994	1.000	0.995
	$R^2$	0.971	0.981	0.978	0.976	0.985	0.972
	$F_{cal}/F_{crit}$	0.860	0.934	0.909	0.866	0.929	0.935
Position	Slope	1.010	1.012	1.011	1.000	1.000	0.999
	$R^2$	0.999	0.999	0.999	0.999	0.999	0.999
	$F_{cal}/F_{crit}$	0.800	0.800	0.801	0.794	0.790	0.796
Acceleration	Slope	0.346	0.914	0.646	0.752	1.387	1.105
	$R^2$	0.935	0.957	0.936	0.915	0.832	0.930
	$F_{cal}/F_{crit}$	0.111	0.637	0.326	0.729	0.932	0.515
Spacing	Slope	0.656	0.585	0.605	1.315	1.356	0.534
	$R^2$	0.594	0.599	0.588	0.703	0.838	0.860
	$F_{cal}/F_{crit}$	7.752	3.089	8.094	1.467	0.849	1.307

The ratio Computed Fisher / Critical Fisher  $\leq 1$  means that one accepts the hypothesis that variances of observed and theoretical values are not statistically different for a level of confidence of 99% ( $\alpha = 0.01$ ;  $D_f = 340$ ).



Overall, all models can reproduce fairly well the behaviour of the cyclist in a following situation. Statistically, all models are significant for reproducing speed, acceleration and position of the following cyclist. However, to estimate the spacing between the two cyclists, only the Raksuntorn's model produced a statistically significant result.

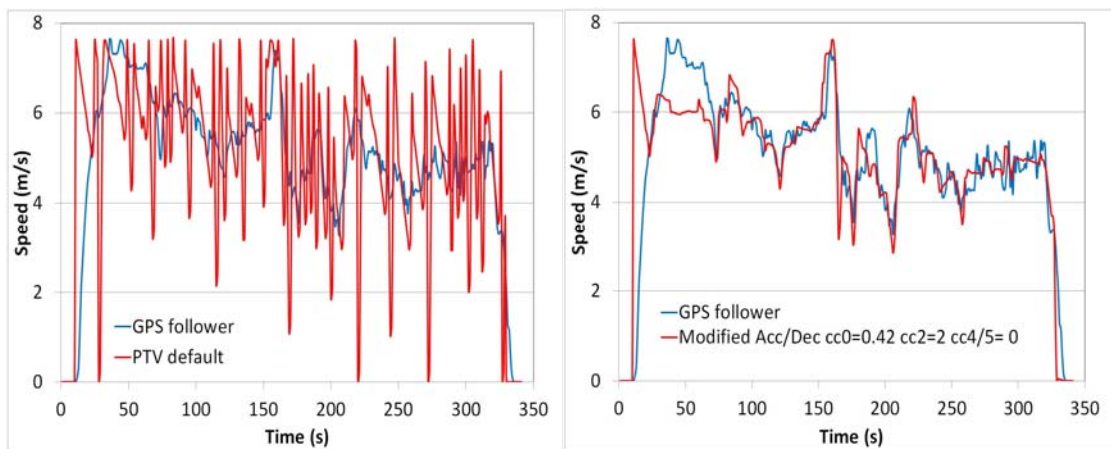
Figure 3 shows the curves for speed (3-a), acceleration (3-b) and spacing (3-c) as modeled by the six bike-following models. These simulated results are compared with the following and leading cyclists' actual profiles obtained through the GPS device. For the case of spacing, it is clear that the models that simulate the speed rather than acceleration produce better results. But when the first 70 seconds during which the follower was trying to catch up the leader are removed (Figure 3-d), all models perform well.



**FIGURE 3 Bike-following models results.**

### Micro-simulation with VISSIM

Moreover, the data collected were used to compare with results produced by micro-simulation software VISSIM (18). During the simulation with a pair of bikes, the leader's positions and speeds were adjusted for each second according to collected data, and the software determine the speeds and acceleration/deceleration of the follower. The experiment was done with default parameters offered with software and with calibrated acceleration/deceleration and driving behaviour parameters. As illustrated in figure 4, the speed of followers produced with default parameters is very sensitive and unstable, but after calibration effort, the result was relatively smooth and convincing. The sum of squared deviation between the observed follower's speeds was reduced from 1051 (with default parameters) to 478 (after calibration).



**FIGURE 4 VISSIM simulation results.**

### BICYCLE TRAFFIC FLOW MODELS

The Highway Capacity Manual 2010 (4) recommends a capacity of 2000 cyclists per hour per lane. However, the capacity values reported in the literature (5) vary from 1500 to 5000 bicycles per hour depending on bicycle facility width. Because traffic flow on bicycle facilities is usually low, capacity can rarely be observed.

The data collected on the off-street facility with video recorder were used to define the fundamental relationships of bicycle traffic. Thus, the flow was calculated from the inverse of headways, and the density from the inverse of spacing.

In order to construct fundamental relationships curves, it was required to determine the theoretical maximum density of the facility. The theoretical maximum density from the experimentation was 0.33 bicycles/m<sup>2</sup>, which is equivalent to 475 bicycles/km for a 1.45 m wide bicycle lane. AASHTO (14) recommends nearly the same value with 0.32 bicycles/m<sup>2</sup>. These values are based on minimal manoeuvring space required for a cyclist.

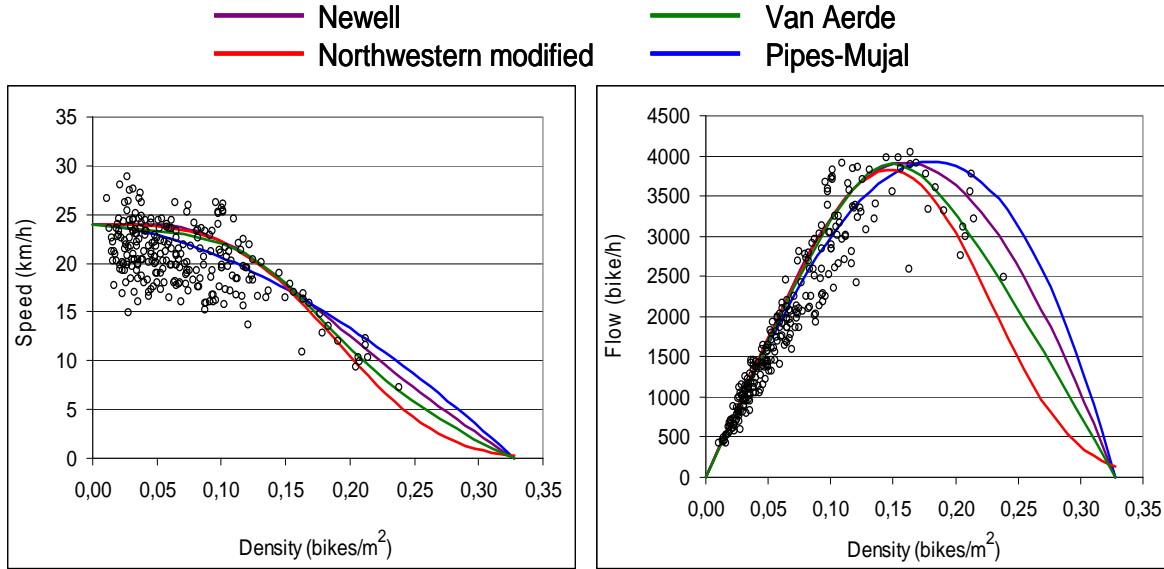
Four existing models were investigated for fit. Table 4 presents the models chosen, the optimized coefficients of their parameters and the R<sup>2</sup> values.

**TABLE 4 Fundamental Relationships Models Investigated**

Models	Formula	Optimized Coefficients & Field Limit Values	R <sup>2</sup> of observed and modeled data values	
			Flow	Speed
Newell (15)	$v = v_f \left[ 1 - e^{-\frac{\lambda}{v_f} (1/k - 1/k_j)} \right]$	$v_f = 24$ $k_j = 475$ $\lambda = 13\ 200$	0.87	0.36
Pipes-Mujal (12)	$v = v_f \left[ 1 - (k/k_j)^n \right]$	$v_f = 24$ $k_j = 475$ $n = 1.65$	0.84	0.35
Northwestern (modified*) (16)	$v = v_f e^{-1/2 (k/k_o)^\alpha}$	$v_f = 24$ $k_o = 250$ $\alpha = 3.4$	0.74	0.31
Van Aerde (17)	$k = - \frac{1}{c_1 + \frac{c_2}{v_f - v} + c_3 v}$	$v_f = 24$ $c_1 = 0.00187135$ $c_2 = 0.00561404$ $c_3 = 0.00010046$	0.83	0.34

\* The constant value of  $\alpha$  in the original model was 2.

These models were calibrated and the resulting curves are presented in figure 5. For the flow-density relationship, all four models reproduced an acceptable fit with observed data. However, the speed-density correlation of all four models with field data is relatively low. This is explained by the high dispersion of speed when bicycle traffic is fluid. This is actually the reflection of differences in physical abilities of cyclists in this sample. In all cases, the shapes of the empirical results are fully consistent with the modeled results.

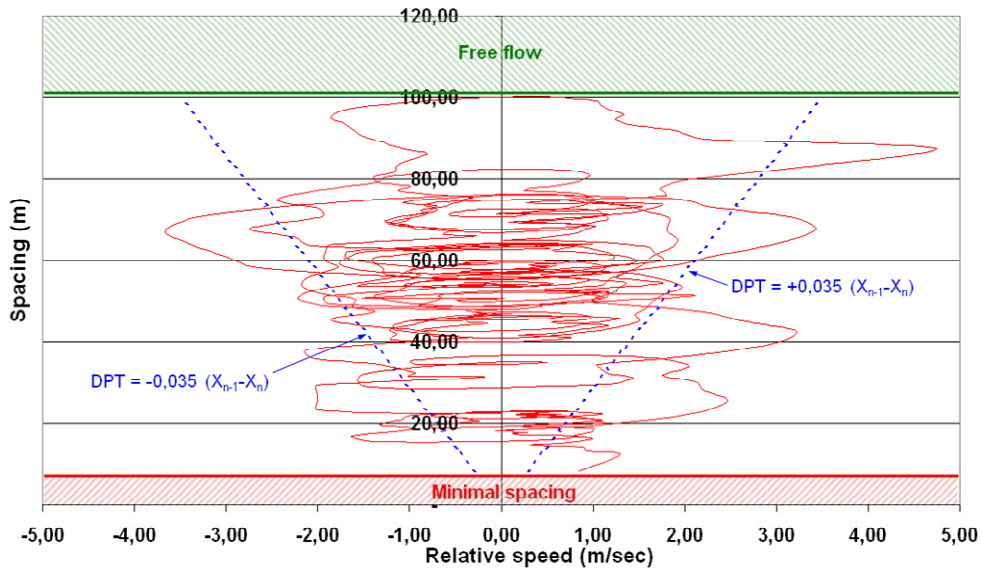


**FIGURE 5 Fundamental relationships comparison with experimental data**

**COMPARISON BETWEEN BIKE-CYCLIST AND CAR-DRIVER SYSTEMS**

To compare the two systems, the same two cyclists drove two cars in a following situation. The same GPS devices and set of rules were used for the bicycle following experiment as for the car following test.

Despite differences in the values of driving parameters for the two systems, the two phase plans (figures 2 and 6) show a similarity in the driving behaviour.

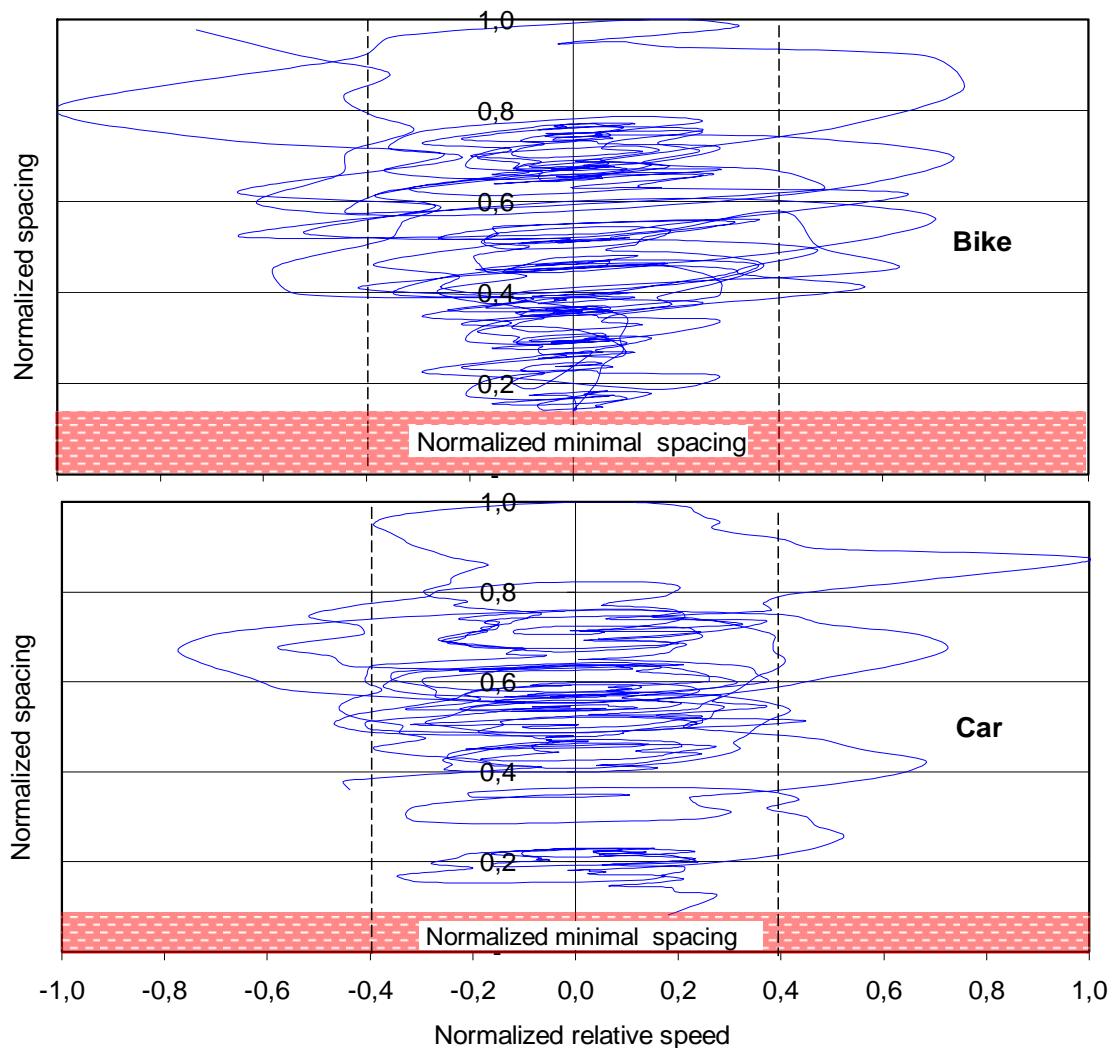


DPT : Distance Perception Threshold

**FIGURE 6 Phase plane trajectory for a pair of cars in following situation**

However, to obtain an objective comparison, a new concept is introduced: a Normalized Phase Plan (Figure 7). The two phase plans are normalized according to the maximum values of each system. Thus, by dividing the values of relative speed for each system by its maximum value, and dividing the values of spacing for each system by its maximum value, the two phase plans obtained can be directly compared.

Figure 7 clearly shows that following drivers, indifferently of the mode used, adopt the same driving logic when seeking the acceptable equilibrium between their desired speed and maintaining a safe distance from the leader. This figure also shows that the amplitude of most phase cycles fall within a 40% deviation of relative speed. It means that, whatever the mode used, the following driver accelerates or decelerates at the same amplitude but not necessarily at the same rate.



**Figure 7 Normalized phases planes trajectories for a pair of cars and bicycles**

## FINDINGS AND CONCLUSIONS

With the increasing popularity of cycling in Montréal, we now see an increased intensity of traffic on bicycle facilities. The authors take advantage of the advances in data gathering technologies to automatically collect data and investigate fundamental as well as bike-following models. In the latter case, a new bike-following model is proposed.

Two types of data were recorded as part of this research. First, a pair of cyclists in a following situation was tracked using a GPS device. Second, a video collection unit was used to film a flow of cyclists on a busy bicycle facility at a fixed spot location. The findings can be summarized as follow:

- There is a distance between two cyclists beyond which the follower moves freely and below which he needs to adjust his speed to the leader to maintain a safe distance when the overtaking is not possible. This zone of influence starts when the spacing is about 16 m. The minimal spacing,  $S_j$ , is 2.2 m including the bicycle length (1.8 m).
- Using the instantaneous flow rate and measuring the space needed by cyclists to manoeuvre, it was found that flow can reach nearly 2700 bicycles/h/m and the maximum density ( $K_j$ ) can reach 0.33 bicycle/m<sup>2</sup> for this 1.45 m width bicycle facility.
- Investigating the fundamental relationships, it was found that the experimental data are fully consistent with the results obtained with theoretical relationships. However, correlation for Speed-Density relationship is relatively low due to greater speed dispersion reflecting the large differences in physical abilities of cyclists. On the other hand, Flow-Density relationships offer a better fit and Newell's model performs better than the others.
- Six bike-following models were calibrated. All models perform well to estimate speed and acceleration, but only Raksuntorn's model is statistically significant for estimating spacing. However, when the first 70 seconds of data are removed, all models estimate well the spacing. These first 70 seconds corresponds to the time the follower took to catch up to the leader. The acceleration values during this time were quite high.
- Using the same experimental conditions, bike-following and car-following observations were compared using normalized phase plans. It can be concluded that a given pair of drivers behave similarly independently of the type of vehicle used.
- Simulation software like VISSIM (18) can be used to simulate bike interaction, but the calibration of parameters is needed to adapt the software to the characteristics of cyclist.

These bike-following models can be use in commercial simulation packages like VISSIM (18), CORSIM (19) or SIMTRAFFIC (20) to account for longitudinal bicycle dynamics. With additional research on cyclist behaviour, and their interaction with other vehicles, the existing simulation tools or the next generation of modelling tools will be able to simulate real world mixed traffic conditions.

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