AN OVERVIEW OF REVISITING THE EMPIRICAL FUNDAMENTAL RELATIONSHIP

Benjamin Coifman, PhD
Associate Professor
The Ohio State University
Joint appointment with the Department of Civil, Environmental, and Geodetic Engineering, and the Department of Electrical and Computer Engineering

Hitchcock Hall 470
2070 Neil Ave, Columbus, OH 43210
Phone: (614) 292-4282
E-mail: Coifman.1@OSU.edu

ABSTRACT
This paper presents an extended overview of a new methodology for deriving an empirical fundamental relationship from vehicle detector data. The new methodology seeks to address several sources of noise present in conventional measures of the traffic state that arise from the data aggregation process, e.g., averaging across all vehicles over a fixed time period. In the new methodology vehicles are no longer taken successively in the order in which they arrived and there is no requirement to seek out stationary traffic conditions; rather, the traffic state is measured over the headway for each individual vehicle passage and the vehicles are grouped by similar lengths and speeds before aggregation. Care is also taken to exclude measurements that might be corrupted by detector errors. The result is a homogeneous set of vehicles and speeds in each bin.

While conventional fixed time averages may have fewer than 10 vehicles in a sample, the new binning process ensures a large number of vehicles in each bin before aggregation. We calculate the median flow and median occupancy for each combined length and speed bin. Then we connect these median points across all of the speed bins for a given vehicle length to derive the empirical fundamental relationship for that length. This use of the median is also important; unlike conventional aggregation techniques that find the average, the median is far less sensitive to outliers arising from uncommon driver behavior or occasional detector errors.

INTRODUCTION
This paper presents an extended overview of Coifman (2014). That paper develops a new methodology for deriving an empirical fundamental relationship (FR) from vehicle detector data. This work is important because much of traffic flow theory depends on the existence of a FR between flow, q, density, k, and space mean speed, v either explicitly, e.g., Lighthill and Whitham (1955), and Richards (1956) (LWR) hydrodynamic model or implicitly, e.g., car following models (Chandler et al., 1958; Gazis et al., 1961). The FR is commonly characterized in terms of a bivariate relationship between two of the three parameters (in each case the third parameter can be calculated from the fundamental equation, repeated in Equation 1). All of the empirically generated FRs use data that average conditions over time and/or space to calculate the traffic state, (q, k, v). It is difficult to measure k directly, so many empirical FR studies use occupancy, occ, as a proxy for k, where: occ is the percentage of the sampling period, T, that
vehicles occupy the detector. As shown in Equation 2, in stationary traffic \( occ \) is proportional to \( k \) by the average effective vehicle length, \( L_{eff} \) during \( T \); where a given vehicle's effective length is the sum of its physical length and the size of the detection zone.

\[
q = k \times v \tag{1}
\]

\[
occ = k \times L_{eff} \tag{2}
\]

Most empirical FR studies use traffic state measurements from conventional detectors that average vehicle measurements over fixed time sampling periods. Figure 1A shows the measured \( q \) versus \( occ \) from one of the dual loop detectors in the Berkeley Highway Laboratory, BHL, (Coifman et al., 2000) on a single lane on a single day, using \( T = 30 \) sec aggregation periods. The results are typical of 30 sec aggregation periods aside from the fact that this location experienced over 12 hrs of recurring congestion on this day, as per the corresponding time series speed shown in Figure 1D. The \( q \) versus \( occ \) plot shows considerable scatter and it is hard to imagine any single curve that would be representative of all of the observed data points. The scatter is commonly attributed to combining non-stationary traffic states, e.g., Cassidy (1998), and debate continues as how best to address the scatter.

The choice of \( T \) is an effort to balance between maximizing the number of vehicles in the sample and minimizing the averaging across inhomogeneous traffic states. Often setting \( T = 30 \) sec is considered to be a good balance between the two competing objectives, though the original use of \( T = 30 \) sec appears to have been for the convenience of telecommunications (Gazis and Foote, 1969). Figure 1B repeats the \( q \) versus \( occ \) for the same vehicles from Figure 1A, only now using \( T = 5 \) min. With the longer sample period, each data point in Figure 1B is more likely to combine different non-stationary traffic states, yet the scatter in this plot is diminished compared to \( T = 30 \) sec. Thus, the noise in the \( T = 30 \) sec plot cannot strictly be due to averaging across non-stationary traffic states. The \( T = 5 \) min plot also exhibits plateauing predicted by Hurdle and Datta (1983) and subsequently illustrated in Hsu and Banks (1993): whereby the \( q \) versus \( occ \) within the queue is truncated at \( q_p \), some maximum \( q \) below the bottleneck capacity, due to traffic entering from on-ramps between the detector location and the bottleneck. This downstream demand consumes a portion of the bottleneck capacity and in this case the resulting plateau in the detector data plot falls around \( q_p = 1,500 \) vph, as shown with a dashed line throughout Figure 1.

Conventional fixed time sampling is a crude methodology that was originally designed to smooth out variability across vehicles in an era when computing power was expensive. While it is true that successive vehicles will usually experience similar traffic conditions while traversing the detector, the impacts of differing vehicle properties can far outweigh any benefits that might come from arbitrarily grouping vehicles based on successive passages, e.g., Coifman (2001) and Coifman et al. (2003) showed empirically that the range of feasible vehicle length undermines conventional relationships between \( q \) and \( occ \). Coifman (in press) used hypothetical microscopic models to revisit the process of generating empirical FR and uncovered several commonly underappreciated factors that result in surprisingly large, non-linear distortions of empirical traffic state measurements.

Briefly reviewing Coifman (in press), in general for a fleet of homogeneous vehicles, stationary traffic, in a sample with a large number of vehicles \( occ \) is related to \( k \) via Equation 2. If one assumes a triangular flow-density FR (denoted \( qkFR \)), the curve is uniquely defined by: capacity, \( q_o \), free speed, \( v_f \), and jam density, \( k_j \). Then, extending to the flow-occupancy FR
(qoccFR), Equation 3 gives jam occupancy, \( \text{occ}_j \). The resulting qoccFR is shown on the right-hand side of Figure 2A for a hypothetical example with \( v_f = 65 \text{ mph}, q_o = 2,400 \text{ vph}, k_j = 211 \text{ vpm} \) and \( L_{\text{eff}} = 20 \text{ ft} \), while the left-hand side shows the corresponding speed-flow relationship (qvFR), transposed from the commonly used orientation to facilitate direct comparisons between these two forms of the FR.

\[
\text{occ}_j = k_j \ast L_{\text{eff}}
\]  

(3)

Even under strictly stationary traffic conditions with homogeneous vehicles, Coifman (in press) found that conventional aggregated \( q, \text{occ} \) and \( v \) measurements should exhibit large scatter in the queued regime arising from a combination of:

(a) errors due to a non-integer number of vehicle headways in a given sample,
(b) averaging over a small number of vehicles during low \( q \),
(c) the inclusion of detector errors, and
(d) the mixing of inhomogeneous vehicles within a sample.

Generally the errors grow larger at lower \( v \). Coifman (in press) also found that, unfortunately, the q-occ plane is skewed such that the noisy low-speed samples cover a disproportionately large area. The points in the qvFR on the left-hand side of Figure 2A are plotted at 5 mph intervals and the corresponding states are shown in the qoccFR by projecting horizontally to the curve on the right-hand side of the figure. Proportionately the qoccFR greatly distorts the relationship to \( v \). The dashed lines in Figure 2A show that when speed has dropped from \( v_f \) by 38\% \( (v = 40 \text{ mph}) \) flow has only dropped from \( q_o \) by 10\%, and when speed has dropped by 85\% \( (v = 10 \text{ mph}) \) flow has only dropped by 49\%. In other words, the higher-speed data are compressed into a narrow sliver, e.g., one can see that the highest speeds in the queued regime are compressed to a small range of \( q \) (or of \( \text{occ} \)); while the low-speed data \( (v < 10 \text{ mph}) \) are spread over more than half of the feasible range of \( q \) or \( \text{occ} \).

**METHODOLOGY**

Coifman (2014) develops a new sampling methodology designed to address the above shortcomings by specifically focusing on the sources of the distortions and minimizing their impacts. In this approach vehicles are no longer taken successively, rather, they are sorted as follows:

1) To eliminate the impacts of non-integer headways, individual vehicle headway, \( h \), is measured instead of \( q \). The associated detector on-time is measured and is used in conjunction with \( h \) instead of \( \text{occ} \). Here \( h \) is measured rear bumper to rear bumper to ensure the driver's chosen gap is combined with their vehicle's on-time.
2) Speed and vehicle length are measured for each individual vehicle.
3) To minimize the impacts arising from a large range of vehicle lengths, the vehicles are sorted into length bins that only span 5 or 10 ft.
4) To minimize the impacts of different speeds, the vehicles are then sorted into speed bins that only span 1 mph.
5) To minimize the impacts of detector errors, any vehicle following an unmatched pulse, involved in a suspected pulse breakup, or following a suspected pulse breakup is excluded because its measured \( h \) may be inaccurate.

6) Finally, to ensure the largest possible number of vehicles per sample, the median headway and median on-time are found for each bin from the subsection of step #3 by step #4 above, and converted to \( q \) and \( \text{occ} \) via Equations 3-4. This clustering approach groups vehicles based on similar speed and length.

\[
q = \frac{1}{\text{median}(h)} \tag{3}
\]

\[
\text{occ} = \frac{\text{median(on)}}{\text{median}(h)} \tag{4}
\]

**ANALYSIS AND DISCUSSION**

The clean hypothetical FR curves in Figure 2A were derived from a homogeneous fleet of vehicles with \( L_{eff} = 20 \text{ ft} \) under stationary conditions, over large sample periods. If the homogeneous fleet has a different \( L_{eff} \), it yields a different curve. To extend the model from Coifman (in press) to homogeneous vehicles with longer \( L_{eff} \), one must recognize that \( q_0 \) and \( k_j \) are functions of \( L_{eff} \). If one assumed that all vehicles came to a stop with a constant physical gap between vehicles, the parameters of \( qk_{FR} \) scale as a function of \( L_{eff} \). Upon using several different values of \( L_{eff} \) to recalculate the FR, it gives rise to a family of curves as shown in Figure 2B. The greater \( L_{eff} \), the lower the curve is in this plot, with the top curve corresponding to \( L_{eff} = 20 \text{ ft} \) and the bottom to \( L_{eff} = 73 \text{ ft} \).

Coifman (2014) applies the process outlined above in the Methodology section to 18 successive days of empirical detector data from one directional detector station in the BHL with four lanes. Combining all of the individual vehicle actuations from all 4 lanes over 18 days at the detector station, measured length is used to sort each vehicle into one of 10 different length bins and individual vehicle speed to further sort these vehicles into one of 70 speed bins. After sorting all vehicles in this manner, Equations 3-4 are applied for each one of these combined length and speed bins. Finally, the \( q\text{occ}_{FR} \) is found by connecting the \( q \) and \( \text{occ} \) values across successive speed bins for a given length bin. The right-hand side of Figure 2C shows the resulting curves by length bin subject to the following three exclusions:

1) To remove the impacts of the downstream inflow, all speeds in all length bins that result in \( q \) above the threshold \( q_p \) for the 18-22 ft length bin are suppressed (as per the corresponding \( qv_{FR} \) in the left-hand side of Figure 2C).

2) To remove the impacts of the measurement errors at low speeds, all speeds below 5 mph in all length bins are suppressed.

3) To remove the impacts of small sample sizes, all combined length and speed bins with fewer than 100 vehicles are suppressed.

As a result of the third exclusion, only seven length bins remain with sufficient data. The seven curves are distinctly visible in the \( q-v \) plane on the left-hand side of Figure 2C. The length range increases from the top curve to the bottom curve in this plot. On the right-hand side the two shortest length bins remain distinct, but the curves from the five longest bins overlap, in part due to the vertical compression as \( L_{eff} \) increases, evident in Figure 2B; in part due to smaller sample sizes in these length bins; in part due to the fact that the speed measurement errors from
exclusion #2 are greater for longer vehicles simply because they are over the detector for a longer amount of time; and in part due to the fact that the actual speed-occupancy relationship appears to exhibit a slight dependency on $L_{eff}$ for these longer vehicles, leading to a small lateral shift in the right-hand side of Figure 2C. All of the curves exhibit trends consistent with the hypothetical example in Figure 2B. Although the speed range on Figure 2C is relatively small, 5 mph to 17 mph, the occ range spans roughly a quarter of the observable values below jam occupancy.

The dotted curves in Figure 2D relax exclusion #1, lifting the upper bound speed exclusion to 50 mph and eliminates the lower bound speed, exclusion #2, altogether (for reference, the curves from Figure 2C are repeated in bold). In the right-hand side of Figure 2D the measured qoccFR curves for most of the length bins flatten out at higher q (corresponding to higher v), i.e., in the region above the threshold $q_p$ from the downstream inflow, shown with a dashed line in these plots. For the shortest length bin (again, accounting for over two thirds of the vehicles) the curve now extends measurements down to 1 mph. The curve remains roughly straight throughout this low speed region, possibly indicating that with a sufficiently large sample size the median is not very sensitive to the outliers arising from the dual loop detector measurement limitations for these very low speeds. If so, the resulting occ range for speeds between 1 and 4 mph covers an additional quarter of the feasible values of occ, with the combined speed range from 1 mph to 17 mph covering more than half of the feasible occ values.

The distortions of the q-occ plane discussed in the context of Figure 2A are readily apparent in Figures 2C-D, as follows. The speed range in Figure 2C spans roughly 18% of the feasible speeds (5 mph ≤ v ≤ 17 mph), but these data span roughly 25% of the feasible q and occ. As shown in Figure 2D, if one extends the range to 1 mph ≤ v ≤ 17 mph (roughly 26% of the feasible speeds) the data now span more than 50% of the feasible q and occ measurements.

CLOSING

This paper provided an extended overview of Coifman (2014). Greater details of the process and discussion of the implications can be found in that paper.

REFERENCES


**FIGURE 1** Conventional q versus occ (A) T = 30 sec, (B) T = 5 min. The corresponding time series (C) flow, and (D) speed. For reference a dashed line is shown at $q_p = 1,500$ vph.
FIGURE 2  (A) Hypothetical qvFR (on the left) and qoccFR (on the right) for a homogeneous fleet of passenger vehicles with $L_{\text{eff}} = 20$ ft under stationary conditions and very long sampling periods, and (B) extending the relationship to homogeneous fleets with longer lengths. The empirical qvFR and corresponding qoccFR for each length bin individually using the data from all 18 days and all four lanes combined together for (C) 5 mph to 17 mph, and (D) 1 mph to 50 mph. Once more, a dashed line is shown at $q_p = 1,500$ vph for reference.