# CALIBRATING MULTILANE FIRST-ORDER TRAFFIC FLOW MODEL WITH ENDOGENOUS REPRESENTATION OF LANE-FLOW EQUILIBRIUM

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## ABSTRACTS

This study develops a multi-lane first-order traffic flow model for freeway networks. In the model, lane changes are considered as a stochastic behavior that an individual driver decrease his/her disutility or cost and are represented as dynamics toward the equilibrium of lane-flow distribution along with longitudinal traffic dynamics. The proposed method can be differentiated from the previous studies in the following points; i) the motivation of changing the lane is explicitly considered, and it is treated as utility defined by current macroscopic traffic state, ii) a whole process of lane-changing is computed by macroscopic manner, that is, the extension of kinematic wave theory employing the IT principle, and iii) in the model framework, lane-flow equilibrium curve will be endogenously generated as a result of self-motivated lane changes. In addition, the parsimony representation enables the parameter calibration by using the data collected from the conventional loop detectors. As a result of the parameter calibration using the data collected at four different sites of Chugoku expressway in Japan, including sag bottleneck, it is revealed that i) the proposed method can represent the lane flow distribution of any observation sites with high accuracy, and ii) the estimated parameters can reasonably explain the multilane traffic dynamics and the bottleneck phenomena on uphill of sag section.

## INTRODUCTION

It is well known that under the condition of high traffic volume lane-flow distribution becomes unbalanced; more traffic tends to use a median lane rather than a middle and outer lane, which causes the deterioration of traffic capacity at bottleneck sections (*1*, *2*, *3*). As intensive development of ITS, active and dynamic lane management has been practically implemented. By employing the technology of ITS, balancing lane-flow distribution is one of the feasible solutions to increase the throughput of bottleneck flow (*3*). Besides the unbalanced lane usage, lane traffic management and control should be considered as one of the solutions to improve the efficiency and safety in case of lane regulation under road works or incidents and at the merging, diverging and weaving sections. For traffic management to be effective, it is needless to say that a model-based decision support system consisting of traffic state estimation, traffic state prediction and optimization and traffic control measures is essential as mentioned in (*4*). However, due to the lack of the method to computing multilane traffic flow including lane-change dynamics, a model-based decision support system enabling lane-based traffic management to be considered has not been realized.

This paper develops the multi-lane first order traffic flow model, which depicts the dynamics of lane-changing. In the model, we assume that each vehicle changes the lane to improve its utility or decrease its disutility, and also that the equilibrium of lane flow distribution is achieved as the condition of stochastic user equilibrium (SUE), where all drivers believe that they cannot improve their utility by changing the lanes anymore. In the model, lane-changes are represented as the dynamics towards lane-flow equilibrium. The utility function for a vehicle to choose each lane is defined by only two parameters on the basis of the investigation about lane-changing behaviour done by Knoop et al. (*5*) and Shiomi et al. (*6*): one is a constant value implying cost breaking the keep-left (or right) rule, and the other one is the average speed depending on the fundamental diagram and the density of the lane. Such parsimony representation enable online calibration by using the real time data from conventional loop detectors. To compute the possible solution of multilane traffic under the conservation law of traffic volume, IT principle (*7*) is applied. Then, in this paper, the parameters reproducing the lane-flow distribution are estimated on the basis of the data collected by the conventional loop detectors. Based on the estimation results, the cross-sectional characteristics of lane-flow distribution at sag section are discussed.

This paper is organized as follow. In section 2, state-of-the-art of modelling multilane traffic is described. In section 3, the concept of lane-change dynamics and the mathematical representation of lane-flow equilibrium are described. In section 4, the computation methods of multilane traffic flow employing IT principle is overviewed. In section 5, the parameter calibration method employing the extended quasi-Newtonian approach is explained, and then in section 6, the application results and discussion are described. Finally, we conclude the contribution of the paper and mentioned the recommendation for the future works.

## STATE-OF-THE-ART

Considerable scientific attention has been paid on the topic of lane-change behaviour and multi-lane flow modelling during the last two decades. Because lane-change is individual vehicle driving behaviour, that is, whether a vehicle changes its lane totally depends on the decision making which the subject vehicle takes and the situation where the subject vehicle is in, it has been the most straightforward way to apply microscopic modelling (*8*, *9*, *10*, *11*, *12*, and more). This approach can consider various conditions and variables which may cause making decision to change lanes. However, due to the computational tasks and complicated model framework, it is not appropriate to apply for online and network-wide freeway traffic evaluation. The other approach is mesoscopic modelling (*13*, *14*). In the approach, gas-kinetic model is applied to depict longitudinal multilane traffic dynamics and lateral movement as well. In Shvestsov and Helbing (*13*), the proportion of lane changers is exogenously given according with the density. The motivations behind the lane change behavior are not appropriately considered. In Hoogendoorn and Bovy (*14*), the probability of a vehicle changing the lane is estimated by applying discrete choice theory. In this case, however, it is required to calibrate various parameters, so that more precise data is required than conventional loop detectors. Also, it is difficult to employ online and dynamic traffic estimation based on the real time data collection.

From the macroscopic approach, Daganzo (*15*, *16*) investigated the traffic phenomena on multilane freeway, and proposed a traffic flow theory based on kinematic wave model, in which it is assumed that there are two types of vehicles; slugs, which have lower desired speed and drive on an outer lane, and rabbits, which have higher desired speed and drive on both outer and inside lanes depending on traffic condition. It was proofed that the slugs and rabbits theory could explain the various traffic phenomena. However, the computational method on the basis of this theory to depict multilane traffic has not been developed. Laval and Daganzo (*7*) proposed a method to computing multilane traffic flow on the basis of kinematic wave theory, and developed a model to depict the influence of the lane-changers to the traffic flow. This study employed the hybrid approach in which lane changers are computed as particles and considered as moving bottlenecks. It is assumed that the number of lane change vehicles is proportional to the differences of traveling speed among lanes. However, it is apparent that this assumption would not represent the lane-flow equilibrium curve appropriately. Besides, the hybrid approach combining microscopic and macroscopic model, which can be seen in Hong et al. (*17*) and Okaue and Okushima (*18*), is not feasible in the model-based decision support system. Tang et al. (*19*), Jin (*20*) and Jin (*21*) developed macroscopic models depicting lane change traffic which considered the disturbances to the traffic flow caused by lane changes. However, the representation of the models is not in lane-specific manner.

This contribution can be differentiated from the previous studies in the following points; i) the motivation of changing the lane is explicitly considered, and it is treated as utility defined by current macroscopic traffic state, ii) a whole process of lane-changing is computed by macroscopic manner, that is, the extension of kinematic wave theory employing the IT principle, and iii) in the model framework, lane-flow equilibrium curve will be endogenously generated as a result of self-motivated lane changes. The proposed model represents a lane-specific traffic dynamics with parsimony manner. Thus, it is expected that it has high feasibility for online and lane-specific traffic state estimation, prediction and evaluation of dynamic lane control scheme.

## MODELING LANE CHANGE AND LANE-FLOW EQUILIBRIUM

### Assumptions of Motivations behind Lane Changes

In this study, we will develop the model depicting lane-changing dynamics and lane-flow equilibrium at a freeway section without any merging and diverging, where all vehicles change the lane to improve their driving circumstances. Namely, mandatory lane-changes heading to off-ramp or coming from on-ramp are not considered.

It is well known that on such section we can observe the specific macroscopic relationship between the total density and the fraction of the lane flow as shown in FIGURE 1. On 2-lanes sections, more traffic tends to use on the outside lane when traffic density is not so large, while under the presence of higher traffic density than approximately 30 [veh/km/2lanes], more traffic drive on the median lane rather than the other lane. As the density increases, the gap of fraction becomes insignificant. On 3-lane sections, it is more complicated rather than 2-lane sections. First, the fraction of the outside lane is more than the other lanes in the presence of less traffic density than 20 [veh/km], and then traffic on the center lane becomes dominant. In the higher traffic density than 50 [veh/km], the fraction of the median lane becomes the largest and finally the gap of fractions among the lanes get insignificant. This tendency is not special to the observation site shown in FIGURE 1, but can be observed generally all over the world.

With regard to the mechanism of the lane-flow distribution and its equilibrium condition, Wu (*2*) theoretically revealed that the equilibrium curve was achieved as a result of balance between the lane-change demand, that is, the proportion of the following vehicles which are force to drive less than their desired speed, and the proportion of the available gap in the adjacent lanes. From the empirical aspects, Knoop et al. (*5*) investigated the relationship between the number of lane-changes and the density of both original and adjacent lanes. It revealed that in the free flow condition the number of lane changes per traffic volume from an outer lane to a median lane and vice versa is not negatively-proportional to the density of the adjacent lane if the density of the original lane is the same level. This fact implies that lane-changes behaviors are not fully explained only by the gap acceptance, that is, the proportion of the available gap. According to Shiomi et al. (*6*) which investigated lane-change behaviors on 3-lanes section by applying a discrete choice model, in the outside and middle lanes, vehicles tend to remain in the original lane, whereas in the median lane, vehicles tend to change lane to either the middle or outer lane. This fact indicates that drivers basically compliant with the keep-left (in case of Japan) rule, which also motivates drivers to change the lane or remain on the same lane.

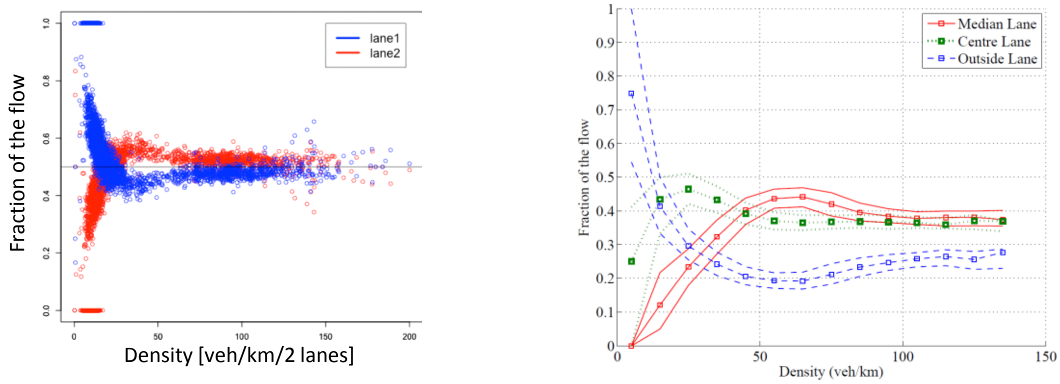
Thus, in this study, it is assumed that:

i) Drivers are motivated to change the lane to increase the driving speed, though it depends on their desired speed; that is, a driver with high desired speed would change the lane and one with low desired speed would not try to that.

ii) Basically, drivers would follow the keep-left rule. That is, if the traffic state is same among lanes, a driver would choose the outside lane.

iii) The demand of lane changes is censored due to the limitation of the available gap on the target lane, which is mutually related to the available capacity of the target lane.

Then, under these assumptions, first order traffic flow model to depict multilane traffic dynamics is developed.




(a) Example of two-lane section

(b) Example of three-lane section (*1*)

**FIGURE 1 Observation of lane-flow distributions in freeways.**

**Definition of Utility Function and Mathematical Expression of Equilibrium State**

Suppose fundamental diagram is defined lane by lane and the average speed of lane $l$, $v_l$, is given as

$$v_l = f_l(k_l),$$

where $f_l(\cdot)$ is a fundamental relationship and $k_l$ is the density on lane $l$, respectively. A driver would choose a lane which gives him/her more utility or less disutility. As mentioned in the previous section, a driver would be motivated to change the lane to increase his/her driving speed or to follow the keep-left rule. Thus, we defined the cost of a vehicle $n$ to drive on lane $l$, $c_{nl}(k_l)$, as a monotonically increasing function against the density:

$$c_{nl}(k_l) = \alpha_l + \beta_l \cdot \{f_l(k_l)\}^{-1} + \varepsilon\,, \tag{1}$$

where $k_l(t, x)$ is the density on lane $l$ at time–space point $(t, x)$, $\alpha_l$ is the disutility to violate the keep-left rule, $\beta_l$ shows sensitivity to the travel time of a unit of distance, and $\varepsilon$ is an error term following Weibull distribution, $W(0,\theta)$, implying the heterogeneity of desired speed and recognition error. Assuming traffic flow is composed of homogenous vehicles in terms of their fundamental diagram and the structure of the cost function, the probability that a vehicle chooses lane $l$ according with the current traffic situation at time $t$ is written as

$$p_l(K) = \frac{\exp[-\theta \cdot c_l(k_l)]}{\sum_k \exp[-\theta \cdot c_k(k_k)]},$$

where $K(t, x)$ is the total density and written as

$$K(t,x) = \sum_l k_l(t,x).$$

Note that an index $n$ is omitted due to the clear representation. The lane-flow equilibrium condition means the state where each driver believes that he/she can no longer decrease the driving cost by changing the lane, or the situation where even if some vehicle change their lanes, other vehicles would compensate for the change of lane traffic flow by changing the lane immediately and as a result lane flow distribution becomes stable. The equilibrium state is indicated by Eq. (2).

$$\begin{aligned} p_l^*(K) &= \frac{\exp\left[-\theta \cdot c_l\left(k_l^*\right)\right]}{\sum_k \exp\left[-\theta \cdot c_k\left(k_k^*\right)\right]} \\ &= \frac{k_l^*}{K}, \end{aligned} \tag{2}$$

where * is the symbol indicating the equilibrium condition.

**Expression of Lane-Change Dynamics**

The equilibrium condition expressed by Eq. (2) is equivalent to the solution of the optimization problem as

$$\min Z(\mathbf{k}) = \sum_k \int_0^{k_k} c_k(\omega) d\omega + \frac{1}{\theta} \sum_k k_k \ln \frac{k_k}{K}$$

subject to [P-1]

$$K = \sum_k k_k$$

$$k_k \geq 0,$$

because the equilibrium condition can be considered as stochastic user equilibrium (SUE) condition, and the cost function is monotonic increasing function with regard to the density. To solve the problem [P-1], the objective function is partially linearized as

$$\min Z(\mathbf{y}) = \sum_k y_k c_k(z_k) + \frac{1}{\theta} \sum_k y_k \ln \frac{y_k}{Y}$$

subject to [P-2]

$$Y = \sum_k y_k$$

$$y_k \geq 0,$$

where $\mathbf{z} = \{z_k\}$ is a vector of the density on lane $k$ at $(t, x)$. Then, the solution vector $\mathbf{y}^*$ is given by calculating KKT condition as

$$y_k^* = Y \cdot \frac{\exp[-\theta \cdot c_k(z_k)]}{\sum_j \exp[-\theta \cdot c_j(z_j)]}. \quad (3)$$

It is mathematically proven that the operation,

$$\mathbf{z}(t + \Delta t, x) = \mathbf{z}(t, x) + (1/\tau)\{\mathbf{y}^* - \mathbf{z}(t, x)\} \quad (4)$$

gives a better solution of [P-1] than $\mathbf{z}(t, x)$, where $\tau > 1$ [Sheffy; 1984]. This result implies that when the cost of each lane is defined as a monotonic increasing function with regard to the density, the lane-flow distribution gradually approaches the equilibrium condition as vehicles repeatedly change lanes in an ad-hoc manner following the choice probability. In this model, the process toward the equilibrium represents the dynamics of lane change. In Eq. (4), an dynamic parameter, $\tau$, is used. It can be interpreted as the same line as $\tau$ of Laval and Daganzo (*7*). Namely, $\tau$ indicates the number of time step a driver takes to decide and execute a lane change. It relates to the gap availability in the target lane. The higher the density of the target lane is, the longer time to find an available gap takes. Thus, $\tau$ is considered as such a parameter that relates to traffic condition, and becomes larger when the density is higher.

## MULTILANE FIRST-ORDER TRAFFIC FLOW MODEL

### Framework of Multilane LWR Model

In this study, multilane LWR model developed by Laval and Daganzo (*7*) is applied with some modification to compute traffic flow on multilane with lane-changes. The conservation law of multilane traffic is written as

$$\frac{\partial K_l(t,x)}{\partial t} + \frac{\partial Q_l(t,x)}{\partial x} = \phi_l, \; l = 1, 2, \ldots, n, \quad (5)$$

where $K_l(t, x)$ and $Q_l(t, x)$ indicate the density and traffic flow on lane l in position x at time t, respectively. The non-homogeneous term, $\phi_l$, in Eq. (5) shows the balance caused by the lane-change vehicles. Thus, this term can be rewritten as

$$\phi_l = \sum_{l' \neq l} \phi_{l' \to l} - \sum_{l' \neq l} \phi_{l \to l'},$$

where $\phi_{l' \to l}$ means the number of vehicles coming from the other lane ($l'$) to the target lane ($l$).

As mentioned above, it is assumed that fundamental diagram is defined lane by lane. Note that lane-change is caused by the differences in the cost among lanes even in the free flow condition. Thus, the equation of fundamental diagram proposed by van Lint et al. (*22*) is used. It is shown as follow.

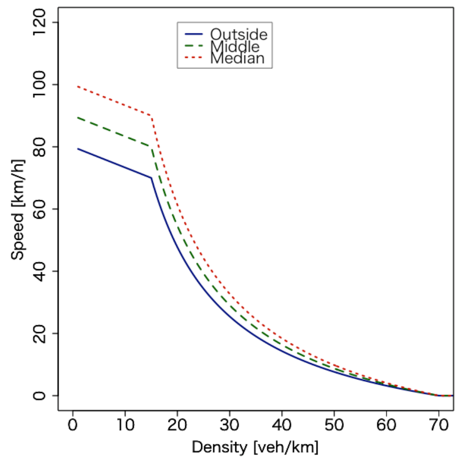



**FIGURE 2 An example of fundamental diagram, where $v_f$ = {80, 90, 100}, $v_c$ = {70, 80, 90}, $k_c$ = {15, 15, 15}and $k_j$ = {70, 70, 70}**.

$$V_l(t,x) = f_l(K_l) = \begin{cases} v_{fl} - K_l \cdot \dfrac{v_{fl} - v_{cl}}{k_{cl}} & if \quad 0 \le K_l \le k_{cl} \\ \dfrac{v_{cl} \cdot k_{cl}}{K_l} \cdot \left(1 - \dfrac{K_l - k_{cl}}{k_{Jl} - k_{cl}}\right) & otherwise \end{cases}, \tag{6}$$

where $v_{fl}$, $v_{cl}$, $k_{cl}$ and $k_{Jl}$ show the free flow speed (km/h), critical speed (km/h), critical density (veh/km) and jam density (veh/km) on lane $l$, respectively. The example of the fundamental diagram following to Eq. (6) is exhibited as FIGURE 2.

**Extension to multilane traffic flow**

*Godunov Scheme for Multilane Section*

To compute the traffic dynamics following the conservation law in Eq. (5), Godunov scheme is applied. Then, Eq. (5) is discretized as

$$\frac{K_{t+1,il} - K_{til}}{\Delta t} + \frac{Q_{til} - Q_{t,i-1,l}}{\Delta x} = \sum_{l' \neq l} \phi_{ti,l' \to l} - \sum_{l' \neq l} \phi_{ti,l \to l'}$$

where the index $t$ and $i$ show time step and cell number, respectively. Following to CFL condition, the time step size $\Delta t$ and cell length $\Delta x$ should keep the constrained condition as follow.

$$\Delta x \ge \max_{\forall l}(v_{fl}) \cdot \Delta t .$$

In the case of single-lane section, the traffic volume transferred from the upper cell $i$ to the downer cell $i$+1, $A_{ti}$, is given as

$$A_{ti} = \min\left(S_{ti}, R_{t,i+1}, (k_{J,i+1} - K_{t,i+1}) \cdot \Delta x\right)$$

*where*

$$S_{ti} = \begin{cases} K_{ti} \cdot V_{ti} \cdot \Delta t & if\, 0 \le K_{ti} \le k_{ci} \\ k_{ci} \cdot v_{ci} \cdot \Delta t & otherwise \end{cases}, \tag{7}$$

$$R_{t,i+1} = \begin{cases} k_{ci+1} \cdot v_{ci+1} \cdot \Delta t & if\, 0 \le K_{t,i+1} \le k_{ci+1} \\ K_{t,i+1} \cdot V_{t,i+1} \cdot \Delta t & otherwise \end{cases}$$

In Eq. (7), $S_{ti}$ is the sending function, which means that the traffic demand from the upper cell, and $R_{t,i+1}$ is the receiving function, which means the supply volume of the downer cell. The transfer volume is limited as the minimum of the traffic demand, supply volume and the physically acceptable number of vehicles in the downer cell. In the following section, the treatment of lane-change vehicles is given.

*Definition of the number of the vehicles with desire of lane-change*

As mentioned in section 3, lane-change vehicles are generated to improve their driving cost. Given the cost function to each lane in accordance with the density of each cell as Eq.(1), the proportion of the vehicles with the desire to change the lane from *l* to *l'*, in time and place (*t*, *x*), $p_{ti,l\rightarrow l'}$, is defined as Eq. (8).

$$p_{ti,l\rightarrow l'} = \frac{\exp[-\theta \cdot c_{il'}(K_{til'})]}{\sum_k \exp[-\theta \cdot c_{ik}(K_{tik})]} \tag{8}$$

Then, the number of vehicles with desire to change the lane is written in accordance with the sending function $S_{til}$ as follow,

$$L_{til\rightarrow l'} = \frac{1}{\tau_{til}} \cdot S_{til} \cdot p_{til\rightarrow l'} ,$$

where $\tau_{til}$ is the dynamics parameter. It is an unknown parameter depending on the traffic state. It is not able to be observed directly so that should be estimated on the basis of the longitudinal variation of lane flow distribution. Along this line, a feedback estimation method (for example, *23*) could be applied to determine the parameter. The volume of the traffic with the desire to keep the lane is also defined as

$$M_{til} = S_{til} - \sum_{l'\in\Omega_l} L_{til\rightarrow l'} ,$$

where Ω shows the set of lanes which a vehicle can get to within time step Δ*t* from the current lane *l*.

*Computing Lane-Change Vehicles*

Based on the number of the vehicles with desire to keep the lane and the number of the vehicles with desire to change the lane, the adjustment process to determine the transfer volume into the downer cells. IT principle (*7*) is applied with partial revisions. In this study, it is assumed that there are two criteria to execute a lane-change. First criterion is whether a vehicle can find a space in the target lane, and the other criterion is whether the downstream cell on the target lane can accept the traffic coming from the upstream of the same lane and its adjacent lanes.

Let $H_{ti+1l}$ denote the total desired number flowing into the cell *i*+1 of lane *l* on time *t* as

$$H_{ti+1l} = M_{til} + \sum_{l'\in\Omega_l} L_{til'\rightarrow l} . \tag{9}$$

For Eq.(9), the first criterion is applied, that is, the desired number of lane-changes is censored according as the acceptable volume on the adjacent cell on the target lane, which makes $H_{ti+1l}$ denoted by

$$H'_{t,i+1,l} = M_{til} + \gamma_{til} \cdot \sum_{l'\in\Omega_l} L_{til'\rightarrow l}$$

*where*

$$\gamma_{til} = \min\left(1, \frac{\min\left(R_{til}, \left(k_{ji} - K_{ti}\right)\Delta x\right)}{H_{ti+1l}}\right)$$

Then, the second criterion is applied. Let $\omega_{til}$ denotes,

$$\omega_{til} = \min\left(1, \frac{\min\left(R_{ti+1l}, \left(k_{ji+1} - K_{ti+1}\right)\Delta x\right)}{H'_{ti+1l}}\right),$$

which defines the possible transfer volume with keeping the lane, $q_{til}$, and the possible transfer volume with lane change, $\phi_{til'\rightarrow l}$, as

$$q_{til} = \omega_{til} \cdot M_{til} ,$$

$$\phi_{til'\rightarrow l} = \omega_{til} \cdot \gamma_{til} \cdot L_{til'\rightarrow l} ,$$

respectively. Then, the actual traffic volume flowing into the downstream cell *i*+1 on lane *l*, $A_{ti+1l}$, is written as

$$A_{ti+1l} = q_{til} + \sum_{l'\neq l} \phi_{til'\rightarrow l} .$$

Finally, the density of each cell is updated every time step in accordance with

$$K_{t+1il} = K_{til} + \left\{ A_{til} - q_{til} - \sum_{l' \neq l} \phi_{til \to l'} \right\} \cdot \Delta x \ .$$

## PARAMETER CALIBRATION METHOD

The proposed model requires the following parameters:

- Fundamental diagrams: free speed, critical density, critical speed, and jam density for each lane
- Utility function of lane-choice: disutility to violate the keep-left rule, and sensitivity to the travel time of a unit of distance for each lane.

In this section, the parameter estimation method on the basis of the data collected by the conventional loop detectors is proposed. Here, the dynamic parameter, $\tau$ in Eq.(4) is excluded in the estimated parameters. It is assumed that the parameters of the utility function of lane-choice are defined under given fundamental diagrams. Thus, the parameters are estimated first for fundamental diagrams, followed by those of utility function of lane-choice.

### Estimation Method of FD Parameters

Conventional loop detectors can directly measure the lane-based traffic volume ($Q_{obs,\ l}$) and space-mean speed ($V_{obs,\ l}$), and the density ($K_{obs,l}$) can be indirectly measured by the operation, $K_{obs,l} = Q_{obs,l}/V_{obs,l}$. Suppose $\mathbf{\Theta}_l$ denotes the unknown parameter vector in FD on lane $l$, the estimation vector $\mathbf{\Theta}_l^*$, which minimizes the residual error between the estimations and the observations, can be found by Eq. (10).

$$\mathbf{\Theta}_l^* = \arg\min_{\Theta_l} \left[ \sum_i^{N_l} \left\{ V_{obs,l}^i - f_l\left(K_{obs,l}^i \middle| \mathbf{\Theta}_l\right) \right\}^2 \right], \tag{10}$$

where $N_l$ shows the number of observations on lane $l$. To solve Eq. (10), quasi-Newton's method is applied.

### Estimation Method of Lane Choice Parameters

For the calibration of lane flow distribution, $\alpha_l$ and $\beta_l$ in Eq. (1) are unknown parameters to be calibrated. According to Eq.(2), it is obvious that only the relative difference of the cost among lanes have influence on the lane choice behavior, so that $\alpha_1$ and $\beta_1$ on the outside lane are given as 0 and 1, respectively. Then, given the total density $K$ and the unknown parameter set $\mathbf{\Phi}$, the proportion of lane flow at the equilibrium condition, $\mathbf{p}_{est}(K|\ \mathbf{\Phi}) = \{p_{est,l}(K|\ \mathbf{\Phi})|\ l = 1, 2, \ldots, n\}$, where $\Sigma_l\ p_{est,l}(K|\ \mathbf{\Phi}) = 1$, is obtained by simulating traffic flow on an imaginary ring road with periodic boundaries. To get the convergence results, the dynamic parameter, $\tau$, is set as the same with the number of time steps in the simulation.

From the conventional loop detectors, the cross-sectional traffic density, $K_{obs}$, and the proportion of lane flow on lane $l$, $p_{obs,l}$, can be observed. It requires huge computational tasks to get the convergence results for all the observed density, so that the estimation of the lane-flow distribution is obtained by the following approximating method. Suppose $\mathbf{x}$ denotes the finite-discrete point sequences shown as $\mathbf{x} = \{x_1, x_2, \ldots, x_n|\ 0 < x_1 < x_2 < \ldots < x_n < K_J\}$. The set of convergence solutions of lane flow equilibrium corresponding to the densities $\mathbf{x}$, $\mathbf{P}_{\mathrm{est}}(\mathbf{x}|\mathbf{\Phi}) = \{\mathbf{p}_{\mathrm{est}}(x_1|\mathbf{\Phi}), \ldots, \mathbf{p}_{\mathrm{est}}(x_1|\mathbf{\Phi})\}$ under given the parameter set $\mathbf{\Phi}$ is computed in advance. Then, the estimate of the proportion of lane flow on lane $l$ corresponding to the observed density, $K_{obs}$, is obtained as a linear interpolation as follow.

$$\hat{p}_{est,l}\left(K_{obs}\middle|\mathbf{\Phi}\right) = \frac{p_{est,l}\left(x_i\middle|\mathbf{\Phi}\right)\cdot\left(x_{i+1} - K_{obs}\right) + p_{est,l}\left(x_{i+1}\middle|\mathbf{\Phi}\right)\cdot\left(K_{obs} - x_i\right)}{x_{i+1} - x_i}, \tag{11}$$

where $x_i \le K_{obs} < x_{i+1}$.

The parameter set $\mathbf{\Phi}$ is found by minimizing the residual errors as follow,

$$\mathbf{\Phi}^* = \arg\min_{\mathbf{\Phi}} \left[ \sum_i^N \sum_l^L \left\{ \hat{p}_{est,l}\left(K_{obs}^i\middle|\mathbf{\Phi}\right) - p_{obs,l}^i \right\}^2 \right], \tag{12}$$

where $L$ is the number of lanes. Eq. (12) can be solved by applying quasi-Newton's method.

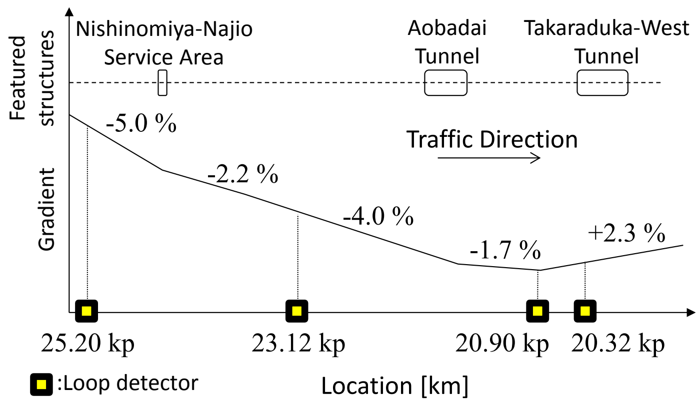



**FIGURE 3 Study section.**

## APPLICATIONS

The proposed method is applied to the real field data. First, the details of the study site and the collected data are described, and the parameter estimation results are shown. Then, the discussion on what the results mean follows.

### Study Section

The data used in the study was collected at 4 cross-sectional points on 3 lanes section of Chugoku Expressway in Japan, which includes the sag section and traffic congestion often occurs behind Takaraduka-West tunnel on 20.32 KP. The geometric features of the target section and the data collection sites are illustrated in FIGURE 3, in which the section between 20.90kp and 20.32kp becomes the bottleneck due to sag structure. The data including 5 min space-mean speed and 5 min traffic volume were collected lane by lane at each observation point on Mar. 16 – 23, Apr. 20 – 30, May. 1 – 18, Jul. 15 – 31, and Sep. 1 – 14, 2010. The data collected under incidents or road works are excluded, so that in total the data of 30 days is used for the further analysis.

### Estimation Results

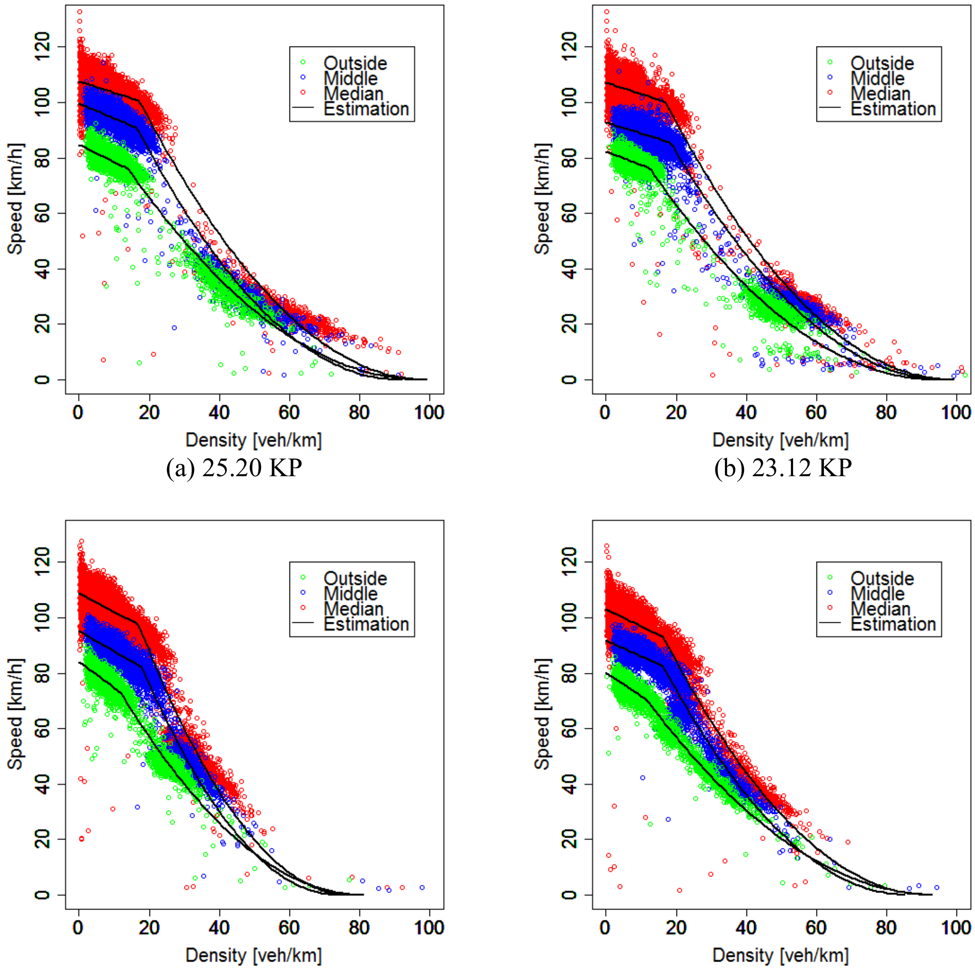
*FD Parameters*

As the first step to calibrate the multilane traffic flow model, the parameters in FD of each lane on each observation site are estimated. The results are summarized in FIGURE 4 and TABLE 1. As seen in FIGURE 4 showing the comparison between the observations and the estimations, the estimates indicate a good fit to the observations except for 25.20 KP, where is located about 5 km upstream from the bottleneck. Interestingly, even in the congested flow region, the average speed on the middle lane is larger than the outer lanes against the same traffic density, and as the density increases the gap of the speed diminishes. This tendency is fully captured by the estimation of FD. Focusing on the traffic capacity, $q_c$, defined as $v_c \cdot k_c$, we can see that traffic capacity at 20.32 KP is much less than the other sites on any lane, which clearly shows 20.32 KP becomes a bottleneck.

*Parameters on Lane Choices*

To estimate the parameters on the cost function for the lane choice in Eq. (1), the variance parameter $\theta$ is empirically set as 1,000. The number of iterations to get the convergence solution of [P-1] is set as 50. The number of cells consists of the imaginary ring road is set as 3. As previously mentioned, at any observation site $\alpha_1$ and $\beta_1$ is set as 0 and 1.0, respectively, to standardize the lane choice costs.

The comparisons between the observations and estimation of lane flow distribution based on the estimated parameters are shown in FIGURE 5, and the estimation results are summarized in TABLE 2. It is interestingly noted that the range of the estimated parameters is almost same for all observation sites. It implies the validity of the estimation results, which can be confirmed by the comparison results showing the good fits to the observations with high accuracy. Concretely, the estimation of the lane flow distribution captures the features that when traffic density is few the fraction of outside lane is dominant, and as the traffic density increases the most dominant lanes shift to the middle lane, and finally the median lane becomes the most dominant, though this feature is slightly different among the observation sites. Thus, these results imply the validity of the approach employed in the research that the lane flow distribution is formulated as the stochastic user equilibrium condition, and the motivations behind the




(a) 25.20 KP (b) 23.12 KP

(c) 20.90 KP (d) 20.32 KP

**FIGURE 4 Comparisons between observations and estimations of FD**

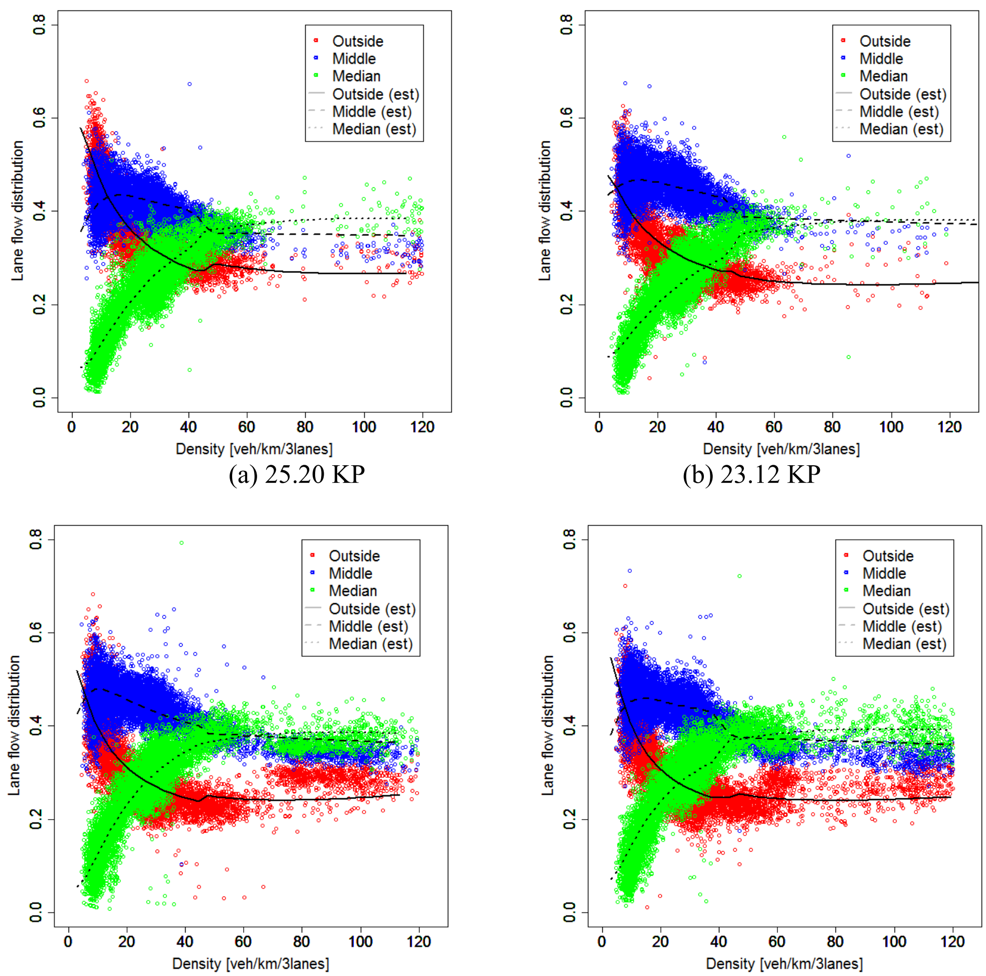
**TABLE 1 FD parameters**

| Site | Lanes | $v_f$ [km/h] | $v_c$ [km/h] | $k_c$ [veh/km] | $q_c$ [veh/h] | $k_J$ [veh/km] |
|---|---|---|---|---|---|---|
| 25.20KP | Outside | 84.7 | 75.9 | 14.1 | 1070.2 | 98.2 |
| | Median | 99.6 | 90.7 | 16.6 | 1505.6 | 91.6 |
| | Middle | 107.6 | 100.6 | 17.2 | 1730.3 | 99.2 |
| 23.12KP | Outside | 82.3 | 76.2 | 12.5 | 952.5 | 94.7 |
| | Median | 92.8 | 85.6 | 18.4 | 1575.0 | 97.1 |
| | Middle | 107.1 | 100.2 | 16.9 | 1693.4 | 99.3 |
| 20.90KP | Outside | 84.1 | 73.0 | 11.9 | 868.7 | 81.5 |
| | Median | 95.2 | 82.1 | 17.9 | 1469.6 | 73.9 |
| | Middle | 108.7 | 97.5 | 17.0 | 1657.5 | 77.2 |
| 20.32KP | Outside | 80.2 | 70.6 | 11.5 | 811.9 | 94.6 |
| | Median | 91.7 | 82.5 | 16.3 | 1344.8 | 86.3 |
| | Middle | 103.1 | 93.0 | 16.3 | 1515.9 | 92.5 |

spontaneous lane changes are simply described by the compliance with the keep-left rule and the sensitivity to the increase of the travel time.

**Discussions**

Hereafter, we discuss the characteristics of lane use along the sag section on the basis of the estimated lane-choice parameters. FIGURE 6 (a) shows the comparisons of the estimated parameter, $\alpha$, indicating the degree of compliance with the keep-left rule. If the average speed of each lane is high, the second term of Eq. (1) becomes lower and the influence of the parameter on lane-choice becomes relatively higher, that is, the parameter, $\alpha$, has more influence in the light traffic situation. The larger this parameter of a lane is, the more the cost to drive on the lane is. According to FIGURE 6(a), it is clearly shown that the value of the median lane is larger than the middle lane for any site, which implies that the median lane is basically not likely to be chosen when traffic volume is not so large. Focusing on the differences among the observation sites, we can see that on 25.20KP, where is the beginning of the downhill section, the parameter of the middle lane is higher than that of 23.12KP, where is the middle point of the downhill section. It means that in the light traffic the fraction of use of the outside lane at 25.20KP is larger and gradually shifts to the middle lane towards the middle point of the downhill at 23.12KP. At



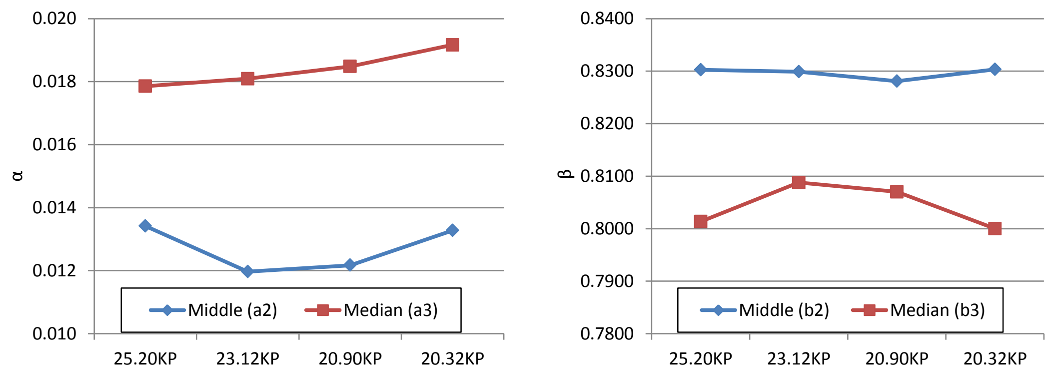

(a) 25.20 KP (b) 23.12 KP

(c) 20.90 KP (d) 20.32 KP

**FIGURE 5. Comparisons between observations and estimations of lane flow distribution**

**TABLE 2 Parameters on lane choices**

| Sites | 25.20KP | 23.12KP | 20.90KP | 20.32KP |
|---|---|---|---|---|
| $\alpha_1$ | 0 | 0 | 0 | 0 |
| $\alpha_2$ | 0.0134 | 0.0120 | 0.0122 | 0.0133 |
| $\alpha_3$ | 0.0179 | 0.0181 | 0.0185 | 0.0192 |
| $\beta_1$ | 1 | 1 | 1 | 1 |
| $\beta_2$ | 0.830 | 0.830 | 0.828 | 0.830 |
| $\beta_3$ | 0.802 | 0.809 | 0.807 | 0.800 |




(a) Parameters of keep-left term (b) Parameters of sensitivity to travel time

**FIGURE 6. Comparisons of estimated parameter values**

the bottom of the sag on 20.90KP, the value of the median lane is slightly higher than 23.12KP, while that of the middle lane is almost same. At the uphill section of 20.32KP, the value of middle lane gets higher as well as the median lane. These facts indicate that in the case of light traffic, on the downhill section where free flow speed tends to be high the proportion of the middle lane becomes high, whereas on the bottom and uphill section, where free flow speed becomes gradually lower, the proportion of lane use shifts to the outside lane. This tendency can be seen in FIGURE 5.

FIGURE 6(b) shows the comparison results of the sensitivity parameter to travel time, $\beta$. This parameter can be interpreted as follows: the larger the parameter is, the less the lane is likely to be chosen as traffic gets congested. It should be interestingly noted that all the estimates shown in FIGURE 6 (b) are less than 1.0, which is corresponding to the parameter of the outside lane. It implies that the outside lane is less likely chosen than the other lanes when traffic volume is high. Also, it can be found that the parameters of the median lanes are less than the middle lanes in any sites, which indicates that the dominant lane shifts from the outside, middle to the median as traffic volume increases. It seems to be valid results. Next, the estimation results are compared among the observation sites. As it can be seen in the figure, the sensitivity of the median lanes on 23.12 KP (at the middle of the downhill) and 20.90 KP (at the bottom of the sag) are higher than the other sites, while no large differences in the sensitivity of the middle lanes among the observation sites are found. It means that on the section from the midpoint of the long downhill to the bottom of the sag, the traffic flow tends to use less on the median lane in comparison to the bottleneck point on 20.32KP. In addition, the fact that the parameter on the middle lane at 20.90 KP is slightly less than the other sites implies that at the bottom of sag traffic flow tends to use more on the middle lane. The underlying mechanism inducing such results can be considered as follow: On the long downhill section, the traveling speed is high enough on average that drivers are not strongly motivated to use the median lane, rather they choose more to drive on the middle lane. When traffic flow comes to the uphill section, it slows down the driving speed on the whole, and drivers on the middle lane try to change the lane to the median to get the speed gain. As pointed out in Patire and Cassidy (*25*), the lateral traffic dynamics beyond lanes might cause speed disturbance, which might makes 20.32 KP a bottleneck. Although these findings are to be confirmed by the direct observations on the lane change behavior, it is revealed that the proposed method can be applied to grasp the characteristics of lane use from the macroscopic point of view.

## CONCLUSIONS

In this contribution, a multi-lane, first-order, macroscopic traffic flow model is developed. In the model, it is assumed that a driver changes the lane to improve the utility or cost of driving circumstance. The utility/cost function is composed by a constant value, indicating the cost to break the keep-left rule, and a coefficient of the inverse of the speed defined by the fundamental diagram, indicating the sensitivity to the increase of the travel time, and an error term, implying the heterogeneity of drivers and the limitation of the information about the surrounding traffic situation. Then, the parameters on the cost function are successfully calibrated on the basis of the data collected from the conventional loop detectors. Also, on the basis of the estimated parameters, the multilane traffic dynamics on the section including sag is discussed. As a result of the parameter calibration using the data collected at four different sites of Chugoku expressway in Japan, including sag bottleneck, it is revealed:

i) the proposed method can represent the lane flow distribution of any observation sites with high accuracy with the observations, and
ii) the estimated parameters can reasonably explain the multilane traffic dynamics and the bottleneck phenomena on uphill of sag section.

In this study, the parameters of the cost function for lane choices are limitedly calibrated. Although these parameters are significant for representing the lane flow distribution, they can limitedly capture the static characteristics of lane use. To depict the occurrence of lane changes, it is essential to calibrate the dynamics parameter, $\tau$, which adjusts the number of lane change vehicles. For this purpose, dynamic feedback system employing Kalman filter family method would be applicable. In addition, it is recommended to upgrade the multilane traffic flow model to depict, for example, mandatory lane changes at such sections with on and off ramps, merging and diverging with multiclass representation.

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