CALIBRATION OF HETEROGENEOUS NONLINEAR CAR-FOLLOWING LAWS FOR TRAFFIC OSCILLATION PREDICTION

Christine Rhoades, Xin Wang, and Yanfeng Ouyang*
Department of Civil and Environmental Engineering at University of Illinois at Urbana-Champaign
205 N Mathews Ave, Urbana, IL 61801
217-333-9858
yfouyang@illinois.edu

INTRODUCTION

Traffic oscillations have been the subject of intensive research since the 1950s. Originally, linear car-following laws were used to analyze oscillation properties [e.g., Chandler et al., 1958; Herman et al., 1958], but more recently, nonlinear car-following laws have been used in order to reproduce more accurate car-following behavior [e.g., Gipps, 1981]. While calibrating the nonlinear laws with field data, however, most research focuses on matching vehicle trajectories in the time domain. Li et al. [2010; 2012] incorporated frequency domain properties (e.g., oscillation periodicity and amplitude) into car-following model calibration such that the describing-function approach (DFA) can be used to predict the oscillation behavior of a homogeneous platoon [Li and Ouyang, 2011]. No consideration was given to ensure prediction accuracy in both the time- and frequency-domains simultaneously.

This paper proposes a new approach to calibrate the parameters in a driver-dependent nonlinear car-following law based on field trajectories. This method is shown to achieve a better balance between time- and frequency-domain trajectory properties. The developed calibration framework implements maximum likelihood estimation with a simulation-based feedback incorporating both time- and frequency-domain prediction errors. The likelihood estimator is obtained from a modified Tobit model to capture the nonlinearity of the car following model (e.g., due to truncation near zero or maximum speeds). The feedback is established by comparing the observed field trajectory with the simulated one under a certain car-following law, where their actual trajectories and frequency spectrums are used to compute their time- and frequency-domain errors, respectively. The car-following law is calibrated for each pair of leader-follower drivers, allowing us to consider a platoon of heterogeneous drivers. The DFA can then be used to predict oscillation propagation in a platoon.

METHODOLOGY

We consider n vehicles, numbered i = 1, 2,…, n, traveling in a platoon in one lane. The trajectory of vehicle i, with position at time t denoted by $x_i(t)$, is assumed to satisfy a nonlinear car-following law. Field trajectory data is used to calibrate the car-following law, and the designed feedback guarantees that the simulated trajectories reproduce the field trajectory well in both the time-space diagram and the frequency spectrum. The DFA can then be used to predict the oscillation propagation for the entire platoon, which serves as a validation of our calibration approach. In the rest of this section, we start by describing the model calibration technique used to obtain simulated trajectories from field data.

Model Calibration

For illustration, we calibrate Newell's nonlinear car-following law, which adjusts the desired velocity based on spacing and physical bounds [Newell, 1961]; i.e.,

$$v_i^*(t) = \text{mid}\{0, k\big(x_{i-1}(t-\tau) - x_i(t-\tau)\big) - \omega, v_{\max}\},$$

where ω is the backward shockwave speed, k is the sensitivity factor of driving aggressiveness, $v_{\max}$ is the upper bound on velocity, and τ is the reaction time. A modified version of the Type I standard Tobit model [Amemiya, 1984] is used to calibrate the vector of model parameters <*k, τ, ω,* $v_{\max}$>. Since velocity has both lower and upper bounds, the Tobit model must be censored from above and below, yielding the following piece-wise model for the actual velocity of vehicle *i* at time *t*:

$$v_i(t) = \begin{cases} 0 & \text{if } v_i^*(t) \le 0 \\ v_i^*(t) + \varepsilon & \text{if } 0 < v_i^*(t) < v_{\max} \\ v_{\max} + \varepsilon & \text{if } v_i^*(t) \ge v_{\max} \end{cases},$$

where the error term, ε, follows a normal distribution with mean zero and standard deviation σ.

The model is then calibrated using maximum likelihood estimation based on observed trajectories in time increments. Note that the velocity upper bound is one of the maximum likelihood estimators (MLE). To obtain a balance between the time- and frequency-domain properties of the trajectory, two adjustments are made to the MLE calculation. The first adjustment ensures accuracy in the simulated trajectory by introducing a penalty based on the mean-square error between the field and simulated trajectories. The second adjustment ensures accuracy in the frequency domain by introducing another penalty based on the difference in amplitude of the two largest frequencies after the discrete Fourier transform is conducted on both the field and simulated trajectories.

After the adjustments are made, a heuristic search method (e.g., simulated annealing) is used to find the best parameter vector <*k, τ, ω,* $v_{\max}$> that maximizes the likelihood function. Since the platoon is made of heterogeneous drivers, the model calibration process is conducted for each pair of consecutive drivers.

Oscillation Prediction

Given the oscillation properties (i.e., amplitude and periodicity) of the leading vehicle, the DFA [Li and Ouyang, 2011] can be applied to predict those of the following vehicle under the calibrated nonlinear car-following law. We consider a general car-following law governing any two adjacent trajectories:

$$x_i(t) = \boldsymbol{L_i}[\boldsymbol{N_i}(x_{i-1}(t) - x_i(t))],$$

where $\boldsymbol{L_i}(\cdot)$ is a linear operator and $\boldsymbol{N_i}(\cdot)$ is a nonlinear function of spacing. The trajectory of each vehicle is first decomposed into its macroscopic, $\bar{x}_i$, and oscillatory, $\hat{x}_i$, components. The Fourier transform is then performed on the oscillatory component, and the describing function method is used to separate the nonlinear and linear components of $\hat{x}_i$, yielding:

$$\hat{x}_i(\Omega) \cong \boldsymbol{L}_i(\Omega) \cdot \boldsymbol{N}_i(\hat{x}_{i-1}(\Omega) - \hat{x}_i(\Omega)) \cdot (\hat{x}_{i-1}(\Omega) - \hat{x}_i(\Omega))$$

in the frequency domain. Figure 1 shows the system block diagram that represents the above equation, where the linear operator contains an integrator and a time delay (i.e., reaction time).

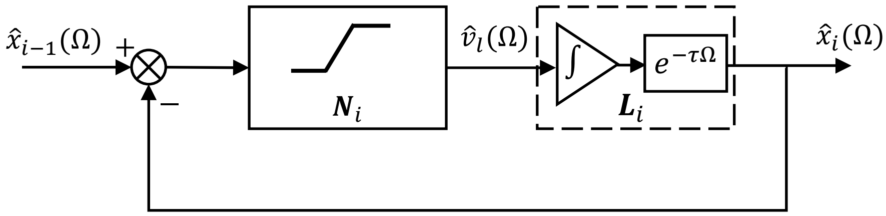

FIGURE 1 Block diagram of Newell's nonlinear car-following law.

Li et al. [2012] proposed a method to find $\hat{x}_i(\Omega)$ for a given $\hat{x}_{i-1}(\Omega)$ in a homogeneous platoon. In this paper, however, the car-following law is calibrated for heterogeneous drivers instead of an "average" driver, so this process must be repeated for each pair of consecutive drivers. The oscillation properties of the trajectories can then be predicted (see Li et al. [2012] for more details) and compared with field trajectories. This confirms the accuracy of the proposed model calibration approach in the frequency domain.

By maintaining a balance between time- and frequency-domain properties, the calibrated car-following models can more accurately reproduce car-following patterns in both domains. The magnitudes of the penalties assigned during the maximum likelihood estimation are adjustable to fit for various scenarios where either the time- or frequency-domain accuracy is more relevant, e.g., higher penalty on frequency domain errors would lead to better oscillation propagation predictions.

NUMERICAL EXAMPLE

Empirical trajectory data are used to validate the proposed model calibration and oscillation prediction framework. For illustration, we consider the platoon consisting of vehicles 86 – 101 from the NGSIM data on southbound US 101 in Los Angeles, CA from 7:50 – 8:35 AM on June 15, 2005.

We use Newell's car-following law for illustration and calibrate the best Newell model parameters for each driver. An example of calibration result (the 92^{nd} driver) can be seen in Figure 2. The calibrated parameters for this driver are <k = 0.7031, τ = 1.0500 sec, ω = 9.5386 ft/s, and $v_{\max}$ = 47.5789 ft/s>.

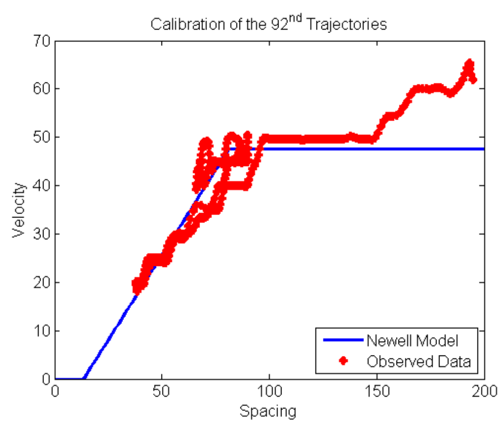

FIGURE 2 Example of Tobit estimation result (driver #92).

With the calibrated car-following model for each driver, trajectories of all vehicles in the platoon can be simulated. The simulated and field trajectories match pretty well in the time domain, as shown in Figure 3. Then, the DFA is performed on the simulated trajectories to obtain the oscillation properties; see Figure 4. It can be seen that the predicted growth of oscillation amplitude matches also quite well with field observation in the frequency domain. More examples and insights will be furnished in the full paper.

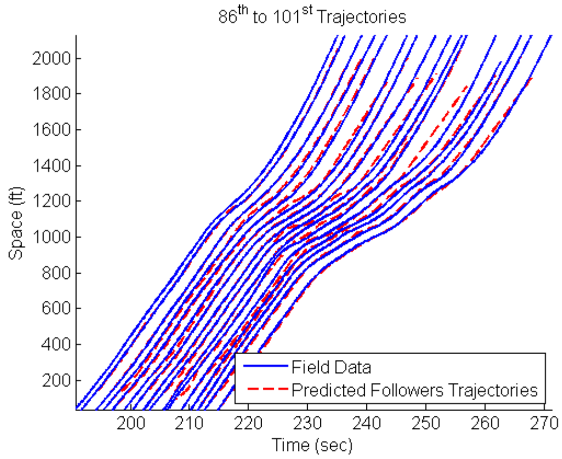

FIGURE 3 Platoon trajectory reproduction in the time domain.

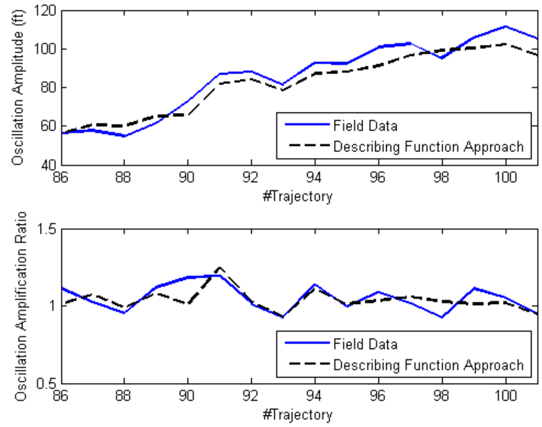

FIGURE 4 Oscillation propagation prediction via frequency-domain DFA.

REFERENCES

Amemiya, T., 1984. Tobit Models: A Survey. Journal of Econometrics 24 (1-2), 3-61.

Chandler, R. E., Herman, R., Montroll, E. W., 1958. Traffic dynamics: Studies in car following. Operations Research 6 (2), 165-184.

Gipps, P. G., 1981. A behavioural car-following model for computer simulation. Transportation Research Part B 15, 105-111.

Herman, R., Montroll, E. W., Potts, R. B., Rothery, R. W., 1958. Traffic dynamics: Analysis of stability in car following. Operations Research 7 (1), 86-106.

Li, X., Peng, F. and Ouyang, Y. (2010) "Measurement and estimation of traffic oscillation properties." Transportation Research Part B, 44(1): 1-14.

Li, X., Ouyang, Y., 2011. Characterization of traffic oscillation propagation under nonlinear car-following laws. Transportation Research Part B 45 (9), 1346–1361.

Li, X., Wang, X., Ouyang, Y., 2012. Prediction and field validation of traffic oscillation propagation under nonlinear car-following laws. Transportation Research Part B 46, 409-423

Newell, G.F., 1961. Nonlinear effects in the dynamics of car following. Operations Research 9 (2), 209–229.