THE HETEROGENEITY OF CAPACITY DISTRIBUTIONS AMONG DIFFERENT FREEWAY LANES

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ABSTRACT

The stochastic nature of highway capacity has drawn increasing attention in recent studies, where the capacity distributions are usually estimated for the whole freeway sections using aggregated data from multiple lanes. However, those cross section-based capacity models cannot be used to assess the risk of semi-congested states observed, in which traffic breaks down on certain lanes while still flowing uninterruptedly on others. This study attempts to investigate the freeway capacity heterogeneity among individual lanes using robust statistical estimation methods. Four diverge sections of different interstate highways in California were selected for the study. Optimal threshold speed was identified for each lane by maximizing the average reduction of traffic efficiency which was defined as the product of mean speed and volume. Capacity observations were obtained for individual lanes and censoring was indicated, based on the optimal threshold speeds. Log-rank and Wilcoxon tests were conducted and the results confirmed the heterogeneity of capacity distributions among lanes of the same freeway section and showed the necessity of estimating capacity distributions for individual lanes separately. A Bayesian hierarchical Weibull model based on censored capacity data was used to estimate these lane-specific capacity distributions. The model parameters are allowed to vary across freeway sections to account for unobserved heterogeneity, and accordingly to improve the accuracy of estimations. In addition, censored data issues are adequately addressed in the proposed model. It is found that breakdown probability would be overestimated if censoring is ignored. The proposed model can provide useful insights when diagnosing bottlenecks with semi-congested cases which may be neglected by the cross section-based models.

Keywords: Lane-specific capacity distribution, semi-congested state, Bayesian hierarchical Weibull model, censored data, optimal threshold speed

INTRODUCTION

Traditional highway capacity is defined as the maximum traffic flow rate that traverses a section under prevailing roadway, traffic and control conditions *(1)*. However, the use of deterministic traffic capacity is an incomplete representation of real-life conditions. The concept of stochastic highway capacity has been recently discussed in several previous studies *(2-4)*, where the capacity is defined as the traffic volume below which traffic still flows and above which the flow breaks down. The capacity in this sense can be regarded as a variable, since the flow rate which causes traffic breakdown is related to a variety of factors such as traffic composition, driving behavior, as well as environmental characteristics *(5)*. The study by Persaud et al. *(6)* on the probabilistic breakdown phenomenon in freeway traffic convincingly showed that that traffic capacity is stochastic in nature.

Regarding the stochastic feature of freeway capacity, it is important to use statistically robust methods to estimate its distribution based on the observed data. Brilon et al. *(4)* applied a nonparametric Kaplan-Meier estimator to obtain breakdown probability based on an analogy with the statistics of survival analysis. Censored observations on capacity were used in their studies. However, a complete capacity distribution function couldn't be obtained using the Kaplan-Meier due to the limitation of data points observed. In the following study by Brilon et al. *(7)*, a parametric method was used to estimate the whole capacity distribution function, where the Weibull model appeared to be the best fit. Recently, Ozguven and Ozbay *(8)* proposed a nonparametric Bayesian survival analysis approach for the estimation of the freeway capacity. Their results indicated that Bayesian estimator could provide robust estimation outputs in the presence of insufficient and unreliable data.

Most of the previous studies *(4, 7, 8)* treated a multi-lane section as a single analysis unit and aggregated data of all the lanes was used to identify breakdowns and estimate cross section-based capacity distributions. However, it should be noted that for multi-lane freeways, the traffic compositions and operational characteristics vary across different lanes. The vehicles with higher speed and better acceleration are more likely to travel on the median lanes. Moreover, the traffic flows of the shoulder lanes have larger chance to be disturbed by merging and diverging vehicles on- and off- ramps. On the other hand, different drivers choose to travel on different lanes according to their destinations *(9)*. In the study by Lawson et al.*(10)*, higher occupancies were observed on the shoulder lanes which was upstream of a congested off-ramp compared with the median lanes. Duret et al. *(11)* affirmed that traffic demand was not assigned evenly among lanes on a freeway section. Therefore, breakdown occurrence can be different even for the lanes of the same freeway section. An empirical study by Dehman and Drakopoulos *(12)* showed the significant difference of breakdown phenomena among lanes. A semi-congested state has also been noted by Munoz and Daganzo *(9)*, where some lanes are congested and other lanes of the same freeway section are not. The application of capacity models at the cross section level will thus ignore the high breakdown probability of certain lanes and cannot accurately assess the risk of semi-congestions. Ma et al. *(13)* proposed a lane-based method to identify breakdown for diverge sections but capacity distributions for specific lanes were not estimated. In addition, censored data and unobserved heterogeneity across different freeway sections were not considered in their study.

This study attempts to investigate the capacity heterogeneity among individual lanes of diverge sections using robust statistical methods. Semi-congestion is a common phenomenon in diverge sections, where the traffic are likely to breakdown on the shoulder lanes connected to the off-ramps, even though the traffic on the other lanes is still flowing. This paper starts with data preparation, where capacity observations and censored data are collected for specific lanes based on optimal threshold speed identified. Hypothesis tests for the heterogeneity of capacity distributions among lanes are conducted to confirm the necessity of developing capacity models at the individual lane level. After that, a novel hierarchical Weibull model is proposed in the Bayesian framework to estimate lane-specific capacity distributions. The parameters of the model are allowed to vary across various sections to account for unobserved heterogeneity. In addition, censored data issues described in *(4)* and *(8)* are explicitly addressed in the proposed model.

DATA PREPARATION

Traffic data used in this study were obtained from the Performance Measurement System (PeMS) of the California Department of Transportation. Four diverge sections which have vehicle detector stations (VDS) right upstream of the off-ramps were selected. These sections have the same lane configuration as shown in FIGURE 1, but they are located in different interstate highways and thus their traffic and environmental features are different. The basic information of the diverge sections selected is presented in TABLE 1. Traffic speed and volume data of the selected sections from January 1st to January 31st in 2014 was extracted for each individual lanes.

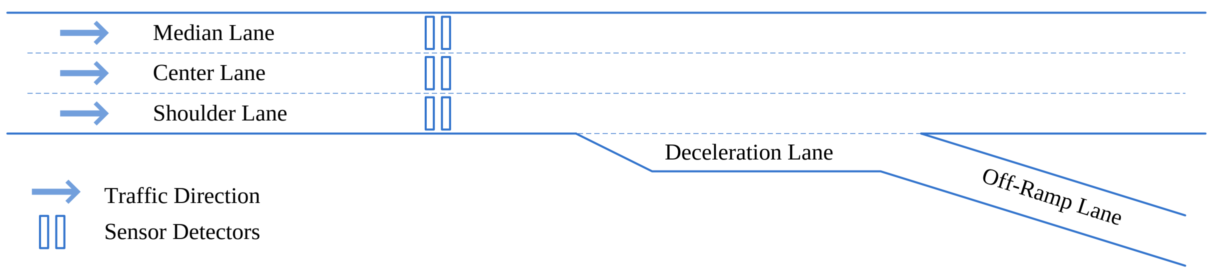


FIGURE 1 Lane configuration and sensor detector location of the four diverge sections selected.

TABLE 1 Overview of the Selected Diverge Sections

	VDS ID	Highway	Post Mile	Direction	County
Section 1	715918	I-5	122.8	North bound	Los Angeles
Section 2	716448	I-105	12.1	West bound	Los Angeles
Section 3	819634	I-15	91.89	North bound	Riverside
Section 4	809070	I-210	61.14	East bound	San Bernardino

Capacity Observations and Censoring

A traffic breakdown is usually followed by a significant amount of reduction in speed. A common method (first proposed in *(4)* and later applied in *(7)*, *(8)* and *(13)*) to identify traffic breakdown points based on mean speed was used in this study. The mean speed (mile/h) and volume (veh/h) were calculated for each lane at 5-minute intervals. The traffic volume of interval i was taken as an observation of capacity if the mean speed starts to fall below threshold speed in the next interval $i+1$.

Censoring refers to a form of imperfect but useful observation of a variable. Censored data is however valuable but requires special consideration to avoid biased estimation results *(14, 15)*. Ignoring censored data could lead to biased estimates of capacity distributions, especially for cases with inadequate breakdown points identified. The identification method of censored data adopted here is consistent with the previous studies such as, *(4)* and *(8)*. If mean speed is higher than the threshold speed in the current interval i and the following interval $i+1$, the volume of the interval i was taken as a right censored observation. A right censored observation provides information that the actual capacity is ensured to be greater than the observed one. As illustrated in FIGURE 2, the volume of the fourth interval is taken as an observation of capacity because the speed starts to be lower than the threshold speed in the following intervals. The volumes in the first three intervals are used as censored observations, since these observed volumes are not high enough to cause breakdowns. Traffic is in the congested state in the last three intervals. As suggested by Brilon *(4)*, the volumes of these intervals don't provide any information for the capacity distribution and thus are excluded from the model development.

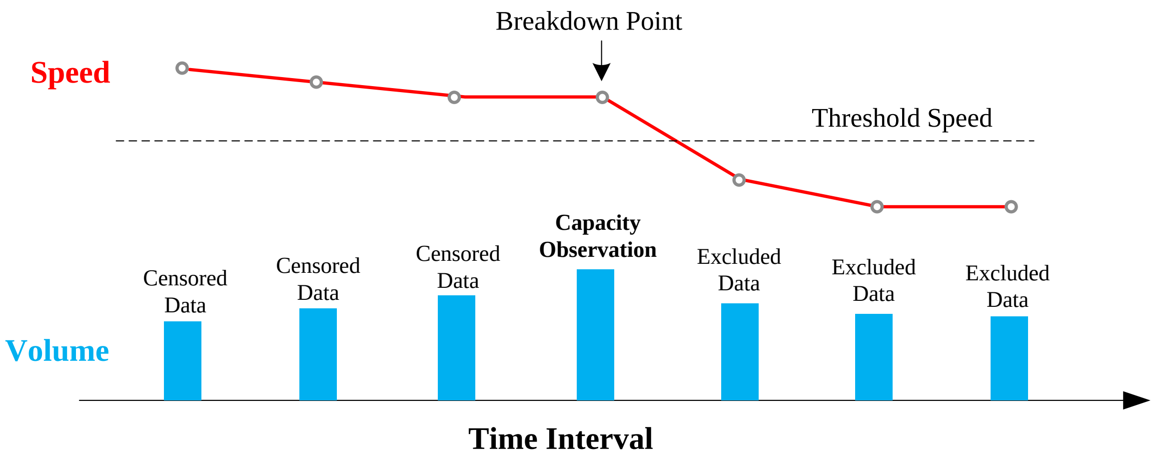


FIGURE 2 Capacity observation and censored data during a breakdown.

Optimal Threshold Speed

In the previous research *(4, 7, 8)*, a standard threshold speed of 70 km/h (about 43.75 mile/h) was used. However, it is undesirable to use the same threshold speed for different lanes, because the threshold speed is likely to be associated with geometric design, traffic composition and environmental features. Ma et al. *(13)* introduced a criterion that can identify the optimal threshold speed by maximizing mean speed reduction and used it to determine threshold speeds for specific lanes. Noting that a traffic breakdown is always accompanied by a significant reduction not only in the traffic speed but also in the traffic volume, we propose to use the product of speed and volume to identify optimal threshold speed in this study.

Let v_i and q_i denote the speed and volume at interval i. We can calculate a measure called as traffic efficiency E_i as the product of v_i and q_i, namely $E_i = v_i \times q_i$. The concept of traffic efficiency was first introduced by Brilon *(16)*. For a certain threshold speed $V_{Threshold}$, if v_i is greater than $V_{Threshold}$ and the speed in the next interval v_{i+1} is lower than $V_{Threshold}$, a breakdown is identified which is accompanied by a reduction of efficiency $\Delta E_i = v_i \times q_i - v_{i+1} \times q_{i+1}$. For all the breakdown points identified using $V_{Threshold}$, the average of efficiency reduction $\overline{\Delta E_i}$ can be computed. After obtaining $\overline{\Delta E_i}$ using a variety of $V_{Threshold}$, the optimal threshold speed $V^*_{Threshold}$ can be identified as the one which leads to the maximum $\overline{\Delta E_i}$.

The aforementioned optimization process is used to compute the optimal threshold speeds for specific lanes and cross sections. As presented in TABLE 2, the average optimal threshold speed for cross sections is 44.00 mile/h, which is quite close to the standard threshold speed of 43.75 mile/h. The data of the median lane in section 4 is used to demonstrate the breakdown points identified by the standard threshold speed (43.75 mile/h) and the optimal threshold speed (52.00 mile/h), as shown in FIGURE 3. The results show that it is more appropriate to use the optimal threshold speed to determine breakdown occurrence since the average of capacity observations identified by the optimal threshold speed (1999 veh/h/lane) is higher than that obtained by the standard threshold speed (1926 veh/h/lane). The optimal threshold speeds listed in TABLE 2 were used to extract the capacity observations and indicate censoring for individual lanes and cross sections.

TABLE 2 Optimal Threshold Speeds for Specific Lanes and Cross Sections (mile/h)

	Median Lane	Center Lane	Shoulder Lane	Cross Section
Section 1	41	35	33	39
Section 2	46	40	37	41
Section 3	49	43	40	48
Section 4	52	47	40	48
Average	47.00	41.25	38.00	44.00

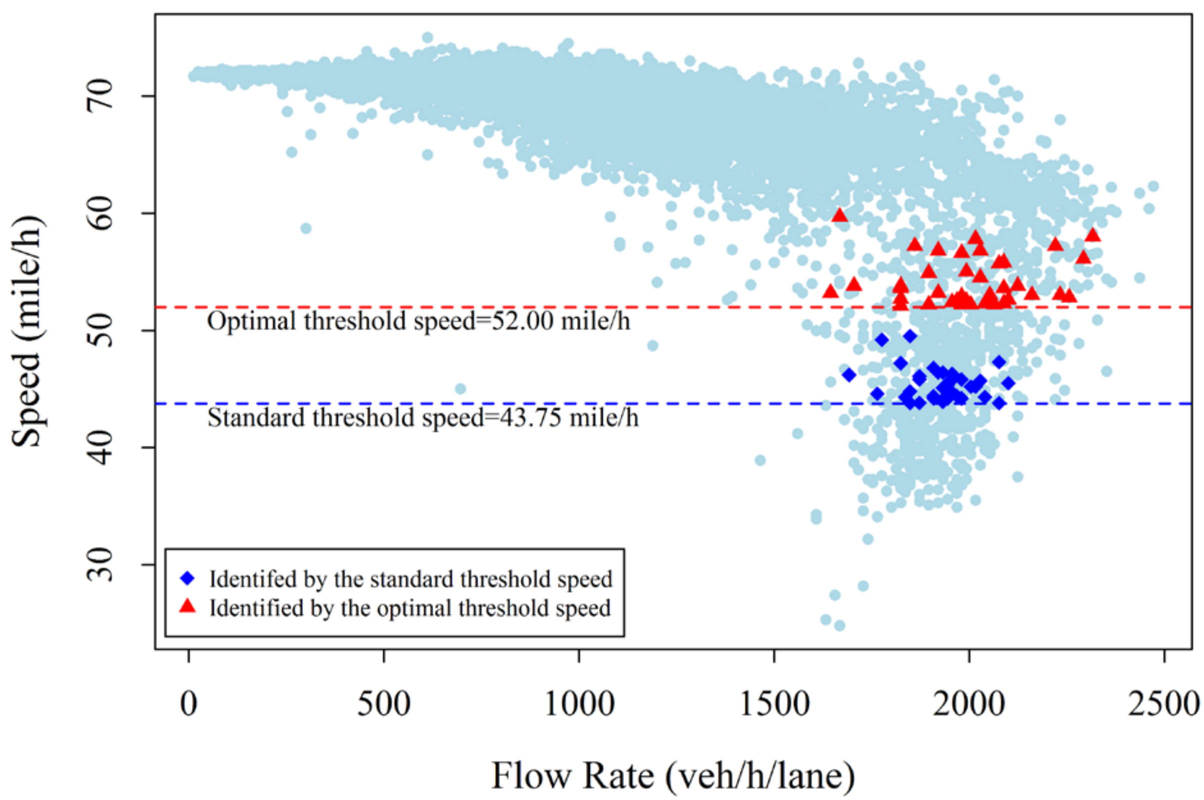


FIGURE 3 Breakdown points identified by standard and optimal threshold speeds (using the median lane of section 4 for demonstration).

HYPOTHESIS TESTS FOR CHECKING CAPACITY HETEROGENEITY

Hypothesis tests for heterogeneity of capacity distributions across lanes were performed using censored capacity data. Let $h_k(q)$ denote the breakdown hazard rate at volume q for the k^{th} lane ($k=1,2...,K$). To compare the capacity distributions among different lanes, the null and alternative hypotheses are set as follows:

H_0: $h_1(q)=h_2(q)=...=h_K(q)$ for all q
H_A: $h_{k_1}(q)\neq h_{k_2}(q)$ for at least one pair of k_1 and k_2, where $1\le k_1\neq k_2\le K$

The volume observations from different lanes were pooled and sorted in ascending order, as follows $q_1\le q_2\le...\le q_D$. At volume q_i, we observed $\mathrm{d_{ik}}$ breakdowns from the k^{th} lane out of Y_{ik} individuals at risk of breakdown, where $i=1,2...,D$. The Nelson-Aalen method *(17)* provides a non-parametric estimator of the hazard rate using censored data. Based on the Nelson-Aalen method, the quantity d_{ik}/Y_{ik} gives an estimate of breakdown hazard rate at volume q_i for the k^{th} lane. It should be noted that the censored observations are supposed to be counted in obtaining Y_{ik}. Similarly, under the null hypothesis, the pooled breakdown hazard rate at volume q_i for the cross section is d_i/Y_i, where $d_i=\sum_{k=1}^{K}d_{ik}$ and $Y_i=\sum_{k=1}^{K}Y_{ik}$. The test of H_0 is based on the weighted difference of estimated hazard rates *(18)*:

$$Z_k=\sum_{i=1}^{D}W_k(q_i)\left(\frac{d_{ik}}{Y_{ik}}-\frac{d_i}{Y_i}\right) \tag{1}$$

where $W_k(q_i)$ is the weight function for the k^{th} lane at volume $\mathrm{q_i}$. In practice, since the commonly used weight functions have the form $W_k(q_i)=Y_{ik}W(q_i)$, we will have the following :

$$Z_k=\sum_{i=1}^{D}W(q_i)\left(d_{ik}-Y_{ik}\frac{d_i}{Y_i}\right) \tag{2}$$

Greater Z_k indicates that the breakdown hazard rate of the k^{th} lane tends to be more different from the expected one under the null hypothesis.

The variance of Z_k and the covariance of Z_{k_1} and Z_{k_2} are given by *(18)*:

$$\sigma_k=\sum_{i=1}^{D}W(q_i)^2\frac{Y_{ik}}{Y_i}\left(1-\frac{Y_{ik}}{Y_i}\right)\left(\frac{Y_i-d_i}{Y_i-1}\right)d_i \tag{3}$$

and

$$\sigma_{k_1k_2}=-\sum_{i=1}^{D}W(q_i)^2\frac{Y_{ik_1}}{Y_i}\frac{Y_{ik_2}}{Y_i}\left(\frac{Y_i-d_i}{Y_i-1}\right)d_i \tag{4}$$

Let Σ represents the estimated $(K-1)\times(K-1)$ variance-covariance matrix of Z_k, where $k=1,2...,K-1$. The overall test statistic for the hypothesis is then defined as follows *(18)*:

$$\chi^2=(Z_1,Z_2,...Z_{K-1})\Sigma^{-1}(Z_1,Z_2,...Z_{K-1})' \tag{5}$$

This overall test statistic is treated as a chi-square distribution with degree of freedom equal to $K-1$. To calculate the χ^2, a variety of weight functions are proposed in the literature. If the $W(q_i)=1$ is selected, it leads to the log-rank test *(19)*, where greater weights are assigned to larger observations. If the weight function is set to be $W(q_i)=Y_i$, it results in the generalized Wilcoxon test *(20)*, where smaller observations are heavily weighted.

Both the log-rank and Wilcoxon tests were conducted to assess the across-lane heterogeneity of capacity distributions for each section separately, based on censored capacity data. As presented in TABLE 3, p-values from both the log-rank and the Wilcoxon tests are less than 0.0001, so the null hypothesis H_0 should be rejected and the alternative hypothesis H_A are accepted. The results indicate significant heterogeneity of capacity distributions among different lanes. To improve the accuracy of assessing breakdown risk, it is critical to develop lane-specific capacity models, which are specified in the next section.

TABLE 3 Results Of Log-Rank and Wilcoxon Tests

	Test	Chi-Square	Degree of Freedom	P-Value	Accepted Hypothesis
Section 1	Log-rank	5180	2	<0.0001	Reject H_0, accept H_A
	Wilcoxon	2616	2	<0.0001	Reject H_0, accept H_A
Section 2	Log-rank	268	2	<0.0001	Reject H_0, accept H_A
	Wilcoxon	71	2	<0.0001	Reject H_0, accept H_A
Section 3	Log-rank	3743	2	<0.0001	Reject H_0, accept H_A
	Wilcoxon	1309	2	<0.0001	Reject H_0, accept H_A
Section 4	Log-rank	45	2	<0.0001	Reject H_0, accept H_A
	Wilcoxon	52	2	<0.0001	Reject H_0, accept H_A

METHODOLOGY OF CAPACITY ESTIMATION

In this section, a full Bayesian approach is proposed to estimate the lane-specific capacity distributions. Bayesian approach provides us with the ability to deal with insufficient data issue, to flexibly select parameter distributions, and to accommodate complicated model structures *(14, 21)*. In Bayesian models, prior distributions are combined with a likelihood function obtained from the observed data to estimate posterior distributions. The Bayesian estimation procedure, prior distributions and assessment of Bayesian models are also introduced in this section.

Bayesian Hierarchical Weibull Model

Let $q=(q_1,q_2,...,q_n)$ denote the traffic volume data. According to the previous research *(7, 13)*, q can be assumed to follow a Weibull distribution $w(\alpha,\lambda)$. The probability density function and cumulative density function of q are given by:

$$f(q\,|\,\alpha,\lambda)=\alpha q^{\alpha-1}\exp(\lambda-\exp(\lambda)q^{\alpha}) \tag{6}$$

and

$$F(q\,|\,\alpha,\lambda)=\int f(q\,|\,\alpha,\lambda)dq=1-\exp(-\exp(\lambda)q^{\alpha}) \tag{7}$$

The survival function $S(q\,|\,\alpha,\lambda)$ gives the probability that traffic keeps flowing until it reaches volume q. The $S(q\,|\,\alpha,\lambda)$ can be derived from $F(q\,|\,\alpha,\lambda)$ as follows:

$$S(q\,|\,\alpha,\lambda) = 1 - F(q\,|\,\alpha,\lambda) = \exp(-\exp(\lambda)q^{\alpha}) \tag{8}$$

To quantify the instantaneous risk that traffic breaks down at volume q, a hazard function $H(q\,|\,\alpha,\lambda)$ is introduced. $H(q\,|\,\alpha,\lambda)$ is defined as the conditional probability of breakdown:

$$H(q\,|\,\alpha,\lambda) = \frac{f(q\,|\,\alpha,\lambda)}{S(q\,|\,\alpha,\lambda)} = \alpha q^{\alpha-1}\exp(\lambda) \tag{9}$$

According to equation (9), if $\alpha > 1$, the breakdown hazard increases as volume q increases; if $\alpha < 1$, the breakdown hazard is negatively associated with volume q; if $\alpha = 1$, the breakdown hazard is not related to volume q.

Let $\nu = (\nu_1, \nu_2, ..., \nu_n)'$ denote the censoring indicators, where $\nu_i = 1$ indicates q_i is a breakdown volume and $\nu_i = 0$ indicates q_i is a censored observation. A censored observation expresses the information that the actual capacity is ensured to be greater than the observed volume. Including the censored observations, the likelihood function of (α,λ) can be written as:

$$\begin{aligned} L(\alpha,\lambda\,|\,q,\nu) &= \prod_{i=1}^{n} f(q_i\,|\,\alpha,\lambda)^{\nu_i} S(q_i\,|\,\alpha,\lambda)^{1-\nu_i} \\ &= \alpha^{d}\exp\{d\lambda + \sum_{i=1}^{n}(\nu_i(\alpha-1)\log(q_i) - \exp(\lambda)q_i^{\alpha})\} \end{aligned} \tag{10}$$

where $d = \sum_{1}^{n}\nu_i$. For the Bayesian Weibull model, it is usually assumed that the parameter α follows a gamma prior distribution $\Gamma(r_0, m_0)$ and λ follows a normal distribution $N(\mu_0, \sigma_0^2)$ *(14)*. The joint posterior distribution of (α,λ) is given by *(14)*:

$$\begin{aligned} &\pi(\alpha,\lambda\,|\,q,\nu) \propto L(\alpha,\lambda\,|\,q,\nu)\pi(\alpha\,|\,r_0,m_0)\pi(\lambda\,|\,\mu_0,\sigma_0^2) \\ &\propto \alpha^{r_0+d-1}\exp\{d\lambda + \sum_{i=1}^{n}(\nu_i(\alpha-1)\log(q_i) - \exp(\lambda)q_i^{\alpha}) - m_0\alpha - \frac{1}{2\sigma_0^2}(\lambda-\mu_0)^2\} \end{aligned} \tag{11}$$

To estimate lane-specific capacity distributions, different sets of Weibull parameters should be assigned to different lanes. In addition, the data structure used in this study can be viewed as a two-level hierarchy with level 1 being the lane level, and level 2 being the freeway section level. Accordingly, a hierarchical model structure is proposed where parameters are allowed to vary to account for the unobserved heterogeneity across sections *(22)*. Weibull parameters $(\alpha_{kj}, \lambda_{kj})$ for the k^{th} lane at the j^{th} section can be specified as follows:

$$\alpha_{kj} \sim \Gamma(r_k, m_k) \tag{12}$$

and

$$\lambda_{kj} \sim N(\mu_k, \sigma_k^2) \tag{13}$$

In equation (6), α_{kj} and λ_{kj} are taken as random parameters which can vary across the sections. Parameters r_k, m_k, μ_k and σ_k^2 are specific for each lane. For the k^{th} lane, the mean and variance of α_{kj} are r_k / m_k and r_k / m_k^2; the mean and variance of λ_{kj} are μ_k and σ_k^2.

Bayesian Estimation Procedure and Priors

Bayesian models are usually estimated via a Markov Chain Monte Carlo (MCMC) algorithm *(23)*. The primary technique of MCMC is Gibbs sampling *(24)*, each iteration of which draws a new value for each unobserved stochastic node from its full conditional distribution given the current values of all the other quantities in the model *(25)*. The WinBUGS statistical software package was used to provide a computing approach for the calibration of Bayesian models using Gibbs sampling *(26)*.

In the absence of credible prior information for model parameters, uninformative priors were used, to express vague and general information about parameters. The priors of r_k and m_k were assumed to be the lognormal distribution (0,10^5); μ_k was assumed to follow the normal distribution (0,10^5); and σ_k^2 was assumed with the Inverse-Gamma distribution (10^{-3}, 10^{-3}). Considering convergence and time of updating, two MCMC chains of 30,000 iterations were run, and the first 10,000 samples were discarded as burn-in. The Brooks-Gelman-Rubin (BGR) diagnostic proposed by *(27)* was used to assess the convergence of multiple chains. Convergence was assumed to occur when the BGR statistic is less than 1.2.

Deviance Information Criterion

The deviance information criterion (DIC) offers a Bayesian measure of model fitting and complexity *(26)*. Specifically, DIC is calculated as follows:

$$DIC = \overline{D(\theta)} + p_D \tag{14}$$

where $\overline{D(\theta)}$ denotes the posterior mean of Bayesian deviance of parameter θ and can be used to indicate how well the model fits the data. p_D defines the effective number of parameters and is taken as a measure of model complexity. Models with smaller DIC are preferred.

RESTULTS AND DISCUSSION

The proposed Bayesian hierarchical Weibull model structure was used to estimate lane-specific capacity distributions. To conduct comparisons, we also developed standard Weibull models where parameters α and λ were fixed for different freeway sections. In addition, to study the impacts of censoring on estimates, two datasets were prepared for modeling: uncensored dataset, where the censored observations on capacity were excluded; and censored dataset, which included censored observations. In view of the long convergence time to estimate Bayesian models with large amounts of data, we sampled the censored observations of the second dataset for the estimation process. The hierarchical Weibull and standard Weibull models were developed based on these two datasets, respectively. Bayesian posterior estimates of parameters and DICs of those models are reported in TABLE 4. $sd(\lambda)$ and $sd(\alpha)$ represent the standard deviations of parameter distributions across sections in the hierarchical models. It should be noted that $sd(\lambda)$ and $sd(\alpha)$ are unavailable for standard Weibull models whose parameters are fixed for different sections. The 95% Bayesian Credible Interval (95% BCI) was used to examine the significance of estimates. Estimates can be regarded as significant at the 95% level if the BCIs do not cover 0 and vice versa *(28)*. Parameter estimates in TABLE 4 are all found to be significant, as their 95% BICs do not cover 0.

TABLE 4 Posterior Summary of Bayesian Models

	Standard Weibull		Hierarchical Weibull	
	Uncensored Dataset	Censored Dataset	Uncensored Dataset	Censored Dataset
	Mean (95% BCI)	Mean (95% BCI)	Mean (95% BCI)	Mean (95% BCI)
Median Lane				
λ	-60.05 (-66.08, -55.64)	-65.89 (-71.46, -59.76)	-76.48 (-83.25, -69.13)	-70.98 (-76.11, -63.67)
$sd(\lambda)$	-	-	0.38 (0.02, 1.59)	0.56 (0.02, 2.65)
α	7.90 (7.33, 8.69)	8.64 (7.83, 9.36)	10.12 (9.09, 11.02)	9.33 (8.28, 10.29)
$sd(\alpha)$	-	-	0.13 (0.07, 0.27)	0.12 (0.04, 0.38)
Center Lane				
λ	-60.09 (-64.92, -55.05)	-64.7 (-70.47, -58.37)	-73.91 (-81.26, -67.48)	-72.73 (-75.79, -67.29)
$sd(\lambda)$	-	-	0.59 (0.02, 3.2)	0.40 (0.02, 1.64)
α	8.04 (7.37, 8.68)	8.61 (7.76, 9.37)	10.00 (8.74, 11.06)	9.75 (8.94, 10.24)
$sd(\alpha)$	-	-	0.17 (0.08, 0.45)	0.11 (0.05, 0.21)
Shoulder Lane				
λ	-44.68 (-49.15, -40.57)	-50.45 (-55.33, -45.4)	-85.6 (-93.42, -76.22)	-75.16 (-80.96, -67.09)
$sd(\lambda)$	-	-	1.29 (0.02, 5.44)	1.36 (0.02, 5.72)
α	5.96 (5.41, 6.55)	6.69 (6.02, 7.33)	11.65 (10.3, 12.7)	10.15 (9.13, 10.96)
$sd(\alpha)$	-	-	0.32 (0.11, 0.62)	00.27 (0.08, 0.54)
Assessment				
$\overline{D(\theta)}$	10132	10543	9610	10120
p_D	6	6	16	17
DIC	10138	10549	9626	10137

Benefits of Hierarchical Structure and Explicitly Addressing Censoring

As shown in TABLE 4, for the uncensored dataset, $\overline{D(\theta)}$ of the hierarchical Weibull model (9610) is 522 less than that of the standard Weibull model (10132). Similarly, based on the censored dataset, the hierarchical Weibull model has a lower $\overline{D(\theta)}$ by 423 in comparison with the standard Weibull model. These findings indicate that the hierarchical Weibull models outperform the standard Weibull models in term of the goodness of fit. In another aspect, higher p_D values are observed in the hierarchical Weibull models. This reflects the increasing complexity as a result of including random parameters. Overall, regarding the lower DICs, the hierarchical Weibull models show substantial improvement although they are penalized by higher p_D values. Note that it is meaningless to compare models using DIC unless they are developed based on the same dataset. So it is erroneous to conclude that the hierarchical model developed based on the uncensored dataset is superior to the one based on censored dataset, although the former has a smaller DIC. The hierarchical models allow parameters α and λ to vary across sections and thus have the flexibility to account for the unobserved heterogeneity. All the estimates of $sd(\lambda)$ and $sd(\alpha)$ are found to be significantly different from 0, and this provides strong evidences for the presence of heterogeneity among sections. By capturing the unobserved heterogeneity, the effects of parameters can be adjusted. As a result of that, the estimates of α and λ of hierarchical Weibull models differ significantly from those of the standard Weibull models. On the other hand, for each section, the same set of parameters is used to duplicate the capacity distributions in the hierarchical

Weibull models, and in this sense, the within-section correlation of capacity observations can be addressed simultaneously.

Using the estimated parameters in TABLE 4 and Weibull cumulative density function shown in equation (7), 25th, 50th and 75th percentiles of capacity distributions were calculated as presented in TABLE 5. Based on the hierarchical Weibull models (uncensored or censored dataset), the 25th, 50th and 75th percentiles of the median lanes are greater than those of the center lanes, respectively. This indicates that greater flow rate is needed for the median lanes than for the center lanes to result in the same breakdown probability. Similarly, as anticipated, the center lanes have higher percentiles compared with the shoulder lanes. However, some inconsistent results (marked by *) are observed when capacity distributions are estimated using the standard Weibull models. The 50th and 75th percentiles of the shoulder lanes are found to be even greater than those of the center lanes, respectively. This result further confirms the advantage of the hierarchical Weibull models over the standard Weibull models in adjusting the parameter estimates. In addition, as shown in TABLE 5, the models developed based censored dataset yield greater percentiles than those excluding censored observations. This finding implies that breakdown probability would be overestimated if censoring is ignored. To get more reliable statistic inferences, the censored observations have to be treated properly.

TABLE 5 The 25th, 50th and 75th Percentiles of the Estimated Capacity Distributions (veh/h/lane)

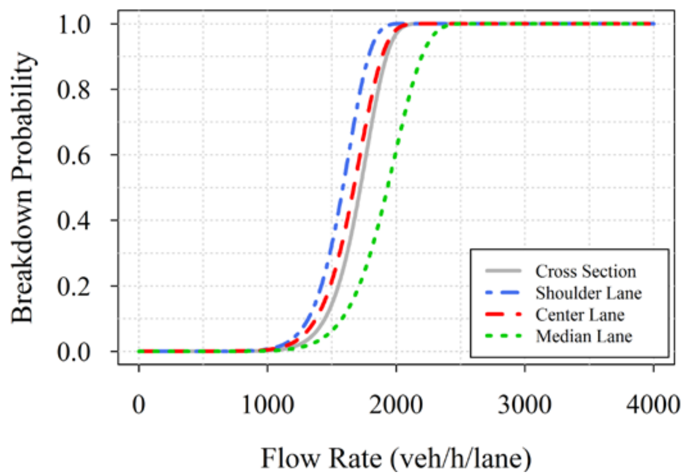
Standard Weibull	Uncensored Dataset			Censored Dataset		
	Median Lane	Center Lane	Shoulder Lane	Median Lane	Center Lane	Shoulder Lane
25th Percentile	1702	1506	1469	1783	1591	1572
50th Percentile	1902	1680	1702*	1974	1762	1793*
75th Percentile	2077	1832	1913*	2139	1910	1989*
Hierarchical Weibull	**Uncensored Dataset**			**Censored Dataset**		
	Median Lane	Center Lane	Shoulder Lane	Median Lane	Center Lane	Shoulder Lane
25th Percentile	1692	1434	1395	1759	1526	1454
50th Percentile	1846	1566	1504	1932	1670	1585
75th Percentile	1977	1678	1596	2082	1794	1697

* Inconsistent results

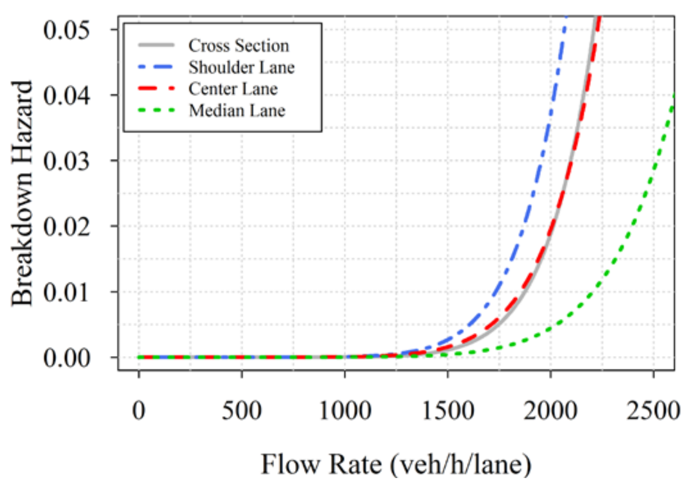
Lane-Specific Capacity Distributions

In view of its superiority, the hierarchical Weibull model developed based on censored dataset was selected to estimate the capacity distributions of specific lanes. As a comparison, the hierarchical Weibull model was also used to estimate the capacity distribution for cross-sections ($\alpha = 10.80$, $\lambda = -80.84$) based on aggregated volume and speed data of multiple lanes. Substituting the estimates of α and λ into the Weibull cumulative density function given in equation (7) and the Weibull hazard function shown in equation (9), breakdown probability and hazard curves for specific lanes and cross sections are plotted. As shown in FIGURE 4, the shoulder lanes have the highest expected breakdown probability and hazard, while the median lanes have the lowest ones, given the same traffic flow rate. According to FIGURE 4a, when the traffic demand is approaching 2000 veh/h/lane, the shoulder and center lanes have extremely high probability to break down, whereas for the median lanes the breakdown probability is approximately 0.6. In FIGURE 4b, all the hazard curves are increasing monotonically, which means that traffic is more likely to break down as the volume increases.

The breakdown probability and hazard curves of the cross sections are close to those of the center lanes, but are significantly different from those of the median and shoulder lanes. It is found that using the cross section-based capacity models for individual lanes would underestimate the breakdown probability and hazard for the shoulder lanes and conversely overestimate them for the median lanes. The lane-based capacity models can be used to estimate the breakdown probability for specific lanes and thus are helpful in improving the accuracy of assessing the congestion risk, especially for the semi-congested cases. However, the high breakdown probability in certain lanes would be ignored by section-based capacity models.



(a)



(b)

FIGURE 4 Breakdown probability and hazard curves for specific lanes and cross sections based on pooled data: (a) breakdown probability, and (b) breakdown hazard.

Using the proposed Bayesian hierarchical model, Weibull parameters for each diverge section can also be estimated. As shown in FIGURE 5, the breakdown probability curves differ from one section to another, and this result is consistent with the significance of $sd(\lambda)$ and $sd(\alpha)$. Moreover, the shoulder lanes have higher $sd(\lambda)$ and $sd(\alpha)$ than those of the median and center lanes according to TABLE 4. This indicates that the capacity distributions of the shoulder lanes vary greatly among sections. It is shown in FIGURE 5 that the lane-specific capacity distributions of section 3 have greater variability than those of others, whereas the capacity distributions of different lanes in sections 2 and 4 tend to be closer to each other. A counterintuitive result is that the breakdown probability of the shoulder lane of section 2 is slightly lower than that of the center lane given the same traffic flow rate. A possible reason is that the section 2 provides proper alignment with the off-ramp, and thus the interaction impacts on the traffic of the shoulder lane may be smaller. It may also be attributed to the different traffic compositions and driving behaviors of specific lanes.

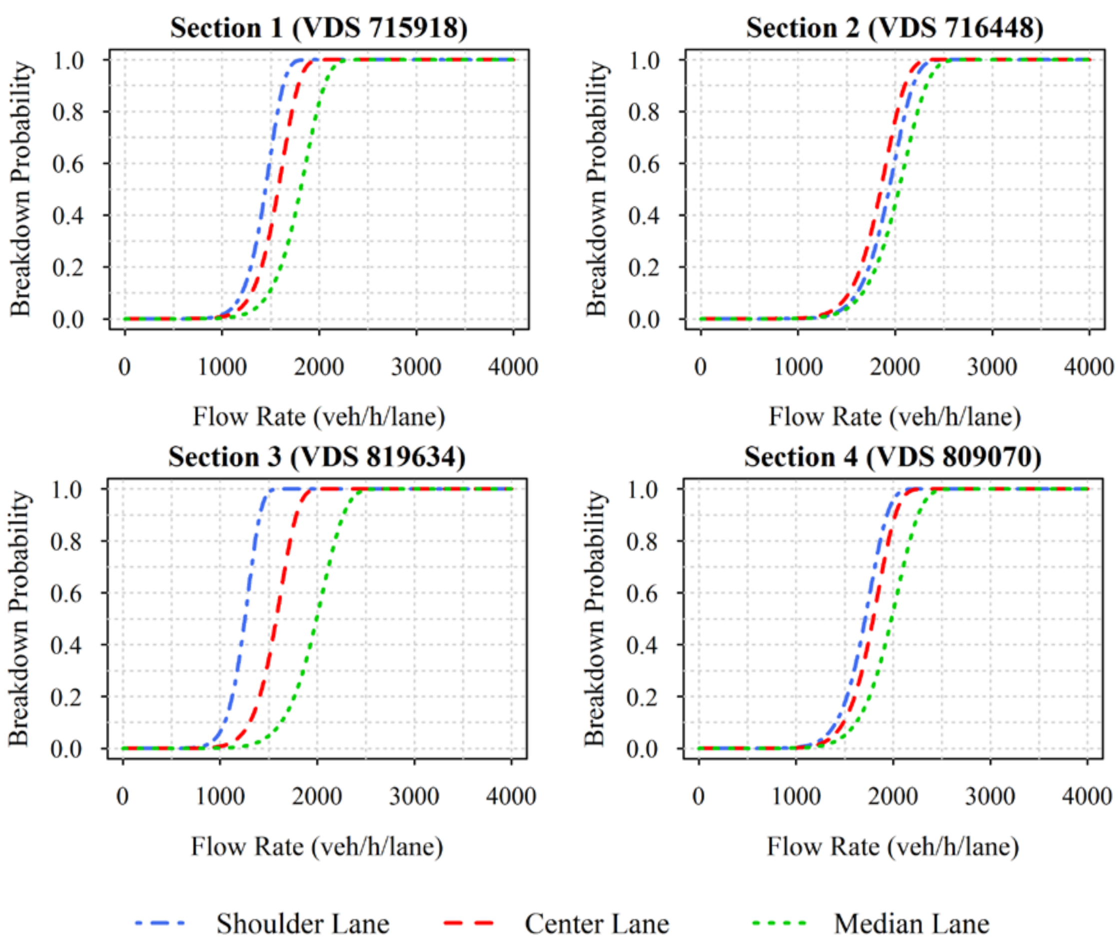


FIGURE 5 Lane-specific breakdown probability curves for each section.

CONCLUSIONS

This study contributes to the literature of stochastic freeway capacity by estimating lane-specific capacity distributions using a novel Bayesian hierarchical Weibull model. Four diverge sections of different interstate highways in California were studied. The speed and volume data of those four sections were retrieved from the PeMS. Noting the difference of traffic features among lanes, a method to obtain the optimal threshold speed by maximizing the average reduction of efficiency was proposed and used to identify traffic capacity observations and indicate censoring. Higher breakdown volumes could be observed by using the optimal threshold speed compared with the standard threshold speed proposed in the literature. Prior to model development, the censored capacity data was examined by the log-rank and Wilcoxon tests. The test results provided strong evidences for the heterogeneity of capacity distributions among individual lanes of the same section and showed the necessity of modeling capacity distributions separately at the lane level. By comparing with other models, the hierarchical Weibull model based on the censored dataset had the best performance and thus was used to investigate the capacity distributions of specific lanes. It is found that breakdown hazard is positively associated with the prevailing traffic volume and lane-specific capacity distributions differ from one section to another. The merits of the proposed model are listed as follows:

- Different sets of Weibull parameters are assumed for the median, center, and shoulder lanes. Such model specification can support lane-based capacity analysis by estimating the breakdown probability for each individual lane.
- The parameters of the hierarchical Weibull model are allowed to vary across freeway sections. By accounting for the across-section unobserved heterogeneity and the within-section correlation simultaneously, the parameter estimates are properly adjusted. In light of its lower DIC value, the proposed model show substantial improvement over the standard Weibull model.
- By integrating the survival analysis theory in the proposed model, the censored data is appropriately treated. It is found that breakdown probability would be overestimated if censoring is ignored.
- A full Bayesian approach is adopted. It improves the proposed model in terms of addressing insufficient data issue, providing flexibility in selecting parameter distributions, and accommodating complicated model structures

The proposed lane-specific capacity model can improve the accuracy of breakdown probability estimation and can provide useful insights when diagnosing bottlenecks with semi-congested cases (i.e. congestions occurring only on certain lanes). Traffic operational performance can be improved by navigating vehicles to choose uncongested lanes to reduce the breakdown occurrence according to the outputs of the lanes-based capacity models. The findings of this paper need further validation using larger data sets from freeway sections of geographic diversity. Additional work to compare Weibull distributions with other types of distributions in the Bayesian hierarchical framework is also suggested. The study on temporal correlation of capacity distributions is also of future interest.

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