Three Influence Regions

Flow vs density

Q₁, Q₂, Q₃, Q₄, L₁, L₂, L₃, L₄
Three Types of q-k Curves
Capacity Drop

![Graph showing flow-density relationship with capacity drop indicated]

- Flow, veh/hr
- Density, veh/km
- Capacity drop highlighted in the graph
Acting Factors Behind?

- **Microscopic View**

  Good Driving Rule (GDR)

  Safe Driving Rule (SDR)

  \[ s^*_{ij} = \tau_i \dot{x}_i + l_j \]

  \[ s^*_{ij} = \frac{\dot{x}_i^2}{2b_i} - \frac{\dot{x}_j^2}{2B_j} + \tau_i \dot{x}_i + l_j \]
SDR can be Aggressive

\[ s_{ij}^* = \frac{x_i^2}{2b_i} - \frac{x_j^2}{2B_j} + \tau_i \dot{x}_i + l_j \]

\[ s_{ij} = \gamma_i \dot{x}_i^2 + \tau_i \dot{x}_i + l_j \]

\[ \gamma_i = \frac{1}{2} \left( \frac{1}{b_i} - \frac{1}{B_j} \right) \]

SDR

Aggressiveness

\[ \gamma_i > 0 \quad \text{Safe Driving Rule} \]

\[ \gamma_i = 0 \quad \text{Good Driving Rule} \]

\[ \gamma_i < 0 \quad \text{Aggressive Driving Rule} \]
Longitudinal Control Model

\[ \ddot{x}_i(t + \tau_i) = A_i \left[ 1 - \left( \frac{\dot{x}_i}{v_i} \right) - e^{1 - \frac{s_{ij}}{s_{ij}^*}} \right] \]

\[ s_{ij}^* = y_i \dot{x}_i^2 + \tau_i \dot{x}_i + l_j \]
Acting Factors Behind?

- **Macroscopic View**

  - Good Driving Rule (GDR)
    \[
    q = \frac{1}{\tau} - \frac{l}{\tau} k
    \]
  - Safe Driving Rule (SDR)
    \[
    q = kv \quad \text{and} \quad k = \frac{1}{\gamma v^2 + \tau v + l}
    \]
  - Longitudinal Control Model (LCM)
    \[
    k = \frac{1}{(\gamma v^2 + \tau v + l)[1 - \ln \left(1 - \frac{v}{v_f}\right)]}
    \]
Influencing Human Factors

- **Microscopic**
  - Desired speed \(v_i\)
  - Effective length \(l_j\)
  - Response time \(\tau_i\)
  - Aggressiveness \(\gamma_i\)

\[
s_{ij}^* = \tau_i \dot{x}_i + l_j
\]
\[
s_{ij}^* = \gamma_i \dot{x}_i^2 + \tau_i \dot{x}_i + l_j
\]
\[
\dot{x}_i(t + \tau_i) = A_i \left[ 1 - \left( \frac{\dot{x}_i}{v_i} \right) - e^{\frac{1-s_{ij}}{s_{ij}}} \right]
\]

- **Macroscopic**
  - Free-flow speed \(v_f\)
  - Average length \(l\)
  - Response time \(\tau\)
  - Aggressiveness \(\gamma\)

\[
q = \frac{1}{\tau} - \frac{l}{\tau} \frac{k}{k}
\]
\[
k = \frac{1}{\gamma v^2 + \tau v + l}
\]
\[
1 = \frac{1}{(\gamma v^2 + \tau v + l)[1 - \ln \left( 1 - \frac{v}{v_f} \right)]}
\]
Influencing Human Factors
Capacity Condition

- **SDR**
  \[ \dot{x}_i = v_i \text{ when } s_{ij} > s_{ij}^* = \gamma_i \dot{x}_i^2 + \tau_i \dot{x}_i + l_j \]
  \[ q_m = \frac{1}{2\sqrt{\gamma l + \tau}} \quad v_m = \sqrt{\frac{\gamma}{l}} \quad k_m = \frac{l}{\gamma^2 + +\tau\sqrt{\gamma l + l^2}} \]

- **GDR**
  \[ \dot{x}_i = v_i \text{ when } s_{ij} > s_{ij}^* = \tau_i \dot{x}_i + l_j \]
  \[ q_m = \frac{v_f}{\tau v_f + l} \quad v_m = v_f \quad k_m = \frac{1}{\tau v_f + l} \]

- **LCM**
  \[ \dot{x}_i = v_i \left( 1 - e^{-\frac{s_{ij}}{s_{ij}^*}} \right) \text{ for all } s_{ij} \text{ where } \dot{x}_i \sim v_i \text{ when } s_{ij} > s_{ij}^* \]

LCM does not yield a closed form of capacity, but can be solved numerically.
Capacity Drop

Flow vs density

- flow, veh/hr
- density, veh/km
Backward Wave Speed

- **SDR**
  \[
  \omega_j = \left. \frac{dq}{dk} \right|_{k=k_j} = \left( \nu - \frac{\gamma \nu^2 + \tau \nu + l}{2\gamma \nu + \tau} \right)_{\nu=0} = -\frac{l}{\tau}
  \]

- **GDR**
  \[
  \omega_j = \left. \frac{dq}{dk} \right|_{k=k_j} = -\frac{l}{\tau}
  \]

- **LCM**
  \[
  \omega_j = \left. \frac{dq}{dk} \right|_{k=k_j} = -\frac{l}{\tau + l/\nu_f}
  \]
## Influencing Human Factors

<table>
<thead>
<tr>
<th>Human Factor Parameters</th>
<th>$v_i$</th>
<th>$l_i$</th>
<th>$\tau_i$</th>
<th>$\gamma_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free-flow speed $v_f$</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Capacity – magnitude $q_m$</td>
<td>N Y Y</td>
<td>Y</td>
<td>Y</td>
<td>Y N Y</td>
</tr>
<tr>
<td>Capacity – location $k_m$</td>
<td>N Y Y</td>
<td>Y</td>
<td>Y</td>
<td>Y N Y</td>
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<td>Capacity – transition</td>
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<td>N N Y</td>
<td>Y</td>
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<tr>
<td>Capacity – drop</td>
<td>N N Y</td>
<td>N</td>
<td>N N Y</td>
<td>N N Y</td>
</tr>
<tr>
<td>Backward wave speed at jam $\omega_j$</td>
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<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Jam density $k_j$</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

Note: Y – has influence; N – no influence. Single letter denotes that the same comment applies to the three models. Three letters (e.g., N N Y) means the comments of SDR, GDR, and LCM, respectively.
### What Do Data Say?

![Graphs showing speed vs density, speed vs flow, flow vs density, and speed vs spacing.](image)

<table>
<thead>
<tr>
<th>Model</th>
<th>$v_f$ (m/s)</th>
<th>$\tau$ (s)</th>
<th>$l$ (m)</th>
<th>$\gamma$ (s²/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe driving rule</td>
<td>29</td>
<td>1.5</td>
<td>6</td>
<td>0.023</td>
</tr>
<tr>
<td>Good driving rule</td>
<td>29</td>
<td>1.5</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>LCM</td>
<td>29</td>
<td>1.3</td>
<td>6</td>
<td>-0.041</td>
</tr>
</tbody>
</table>
Questions? Comments?

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