Heterogeneity of Capacity Distributions among Different Freeway Lanes

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New Concept of Capacity

• Constant Capacity (traditional concept)
  – The maximum traffic flow rate that traverses a section under prevailing roadway, traffic and control conditions. (HCM 2000)

• Stochastic Capacity (new concept)
  – The traffic flow rate which causes traffic breakdown. (Brilon et al. 2005)
  – Becomes a random variable and is related to traffic composition, driving behavior, as well as environmental characteristics.
Motivation & Objective

• Previous Research
  – Treated a multi-lane freeway section as a single analysis unit.
  – Aggregated data from multiple lanes was used.

• Semi-Congested State
  – Traffic compositions and operational features vary across different lanes.
  – Traffic demand was not assigned evenly among lanes.

• Objective
  – Investigate the heterogeneity of capacity distributions among individual lanes.
Data Preparation

- **Data Source**
  - PeMS of the California DOT
- **Sampling**
  - Four diverge sections
  - Interstate highway
  - Similar lane configurations

Lane configuration and sensor detector location
Data Preparation

- **Lane-Level Data**
  - Mean speed and volume were obtained for each individual lane at 5-min intervals

- **Capacity Observations and Censored Data**

  ![Graph showing speed and volume over time with breakdown and threshold points]

  - Breakdown Point
  - Threshold Speed

  **Capacity observation and censored data**
Data Preparation

• Process to Determine Optimal Threshold Speed

- Traffic efficiency $E$ is defined as the product of speed $v$ and volume $q$.
- Determine the optimal threshold speed that can maximize the average of efficiency reduction ($E_i - E_{i-1}$).
Data Preparation

- Optimal Threshold Speed Identified

<table>
<thead>
<tr>
<th>Section</th>
<th>Median Lane</th>
<th>Center Lane</th>
<th>Shoulder Lane</th>
<th>Cross Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section 1</td>
<td>41</td>
<td>35</td>
<td>33</td>
<td>39</td>
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<tr>
<td>Section 2</td>
<td>46</td>
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<td>41</td>
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<td>Section 3</td>
<td>49</td>
<td>43</td>
<td>40</td>
<td>48</td>
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<tr>
<td>Section 4</td>
<td>52</td>
<td>47</td>
<td>40</td>
<td>48</td>
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<tr>
<td>Average</td>
<td>47.00</td>
<td>41.25</td>
<td>38.00</td>
<td>44.00</td>
</tr>
</tbody>
</table>

Average capacity identified by the optimal threshold speed is 1999 veh/h/lane

Average capacity identified by the standard threshold speed is 1926 veh/h/lane

Use the median lane of section 4 for demonstration
Tests for Capacity Heterogeneity

• Null and Alternative Hypotheses

\[ H_0: \ h_1(q) = h_2(q) = \ldots = h_K(q) \ \text{for all} \ q \]
\[ H_A: \ h_{k_1}(q) \neq h_{k_2}(q) \ \text{for at least one pair of} \ k_1 \ \text{and} \ k_2, \]
where \( 1 \leq k_1 \neq k_2 \leq K \)

• Test Statistics

For \( k \text{th lane}: \)
\[ Z_k = \sum_{i=1}^{D} W_k(q_i) \left( \frac{d_{ik}}{Y_{ik}} - \frac{d_i}{Y} \right) \]

For all the lanes: \[ \chi^2 = (Z_1, Z_2, \ldots Z_{K-1}) \Sigma^{-1} (Z_1, Z_2, \ldots Z_{K-1})' \]

The overall test statistic \( \chi^2 \) is treated as a chi-square distribution with degree of freedom \( K-1 \)
Tests for Capacity Heterogeneity

• Log-Rank Test and Wilcoxon Test
  – If $W(q_i)=1$, it leads to the log-rank test
  – If $W(q_i)=Y_i$, it leads to the Wilcoxon test

<table>
<thead>
<tr>
<th>Section</th>
<th>Test</th>
<th>Chi-Square</th>
<th>Degree of Freedom</th>
<th>P-Value</th>
<th>Accepted Hypothesis</th>
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<tbody>
<tr>
<td>1</td>
<td>Log-rank</td>
<td>5180</td>
<td>2</td>
<td>&lt;0.0001</td>
<td>Reject $H_0$, accept $H_A$</td>
</tr>
<tr>
<td></td>
<td>Wilcoxon</td>
<td>2616</td>
<td>2</td>
<td>&lt;0.0001</td>
<td>Reject $H_0$, accept $H_A$</td>
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<tr>
<td>2</td>
<td>Log-rank</td>
<td>268</td>
<td>2</td>
<td>&lt;0.0001</td>
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<tr>
<td></td>
<td>Wilcoxon</td>
<td>71</td>
<td>2</td>
<td>&lt;0.0001</td>
<td>Reject $H_0$, accept $H_A$</td>
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<tr>
<td>3</td>
<td>Log-rank</td>
<td>3743</td>
<td>2</td>
<td>&lt;0.0001</td>
<td>Reject $H_0$, accept $H_A$</td>
</tr>
<tr>
<td></td>
<td>Wilcoxon</td>
<td>1309</td>
<td>2</td>
<td>&lt;0.0001</td>
<td>Reject $H_0$, accept $H_A$</td>
</tr>
<tr>
<td>4</td>
<td>Log-rank</td>
<td>45</td>
<td>2</td>
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<td>Reject $H_0$, accept $H_A$</td>
</tr>
<tr>
<td></td>
<td>Wilcoxon</td>
<td>52</td>
<td>2</td>
<td>&lt;0.0001</td>
<td>Reject $H_0$, accept $H_A$</td>
</tr>
</tbody>
</table>
Bayesian Hierarchical Weibull Model

- **Weibull Distribution**
  - The breakdown volume $q$ is assumed to follow Weibull distribution $W(\alpha, \lambda)$

  **Probability density function:**
  $$f(q \mid \alpha, \lambda) = \alpha q^{\alpha - 1} \exp(\lambda - \exp(\lambda)q^\alpha)$$

  **Cumulative density function:**
  $$F(q \mid \alpha, \lambda) = \int f(q \mid \alpha, \lambda) dq = 1 - \exp(-\exp(\lambda)q^\alpha)$$

  **Survival function:**
  $$S(q \mid \alpha, \lambda) = 1 - F(q \mid \alpha, \lambda) = \exp(-\exp(\lambda)q^\alpha)$$

  **Hazard function:**
  $$H(q \mid \alpha, \lambda) = \frac{f(q \mid \alpha, \lambda)}{S(q \mid \alpha, \lambda)} = \alpha q^{\alpha - 1} \exp(\lambda)$$

- **Address Censoring Issues**
  - Likelihood function

  $$L(\alpha, \lambda \mid q, \nu) = \prod_{i=1}^{n} [f(q_i \mid \alpha, \lambda)^{\nu_i} S(q_i \mid \alpha, \lambda)^{1-\nu_i}]$$

  $$= \alpha^d \exp\{d\lambda + \sum_{i=1}^{n} (\nu_i(\alpha - 1)\log(q_i) - \exp(\lambda)q_i^\alpha)\}$$

  - Probability density function of capacity observation
  - Survival function of censored data
Bayesian Hierarchical Weibull Model

- Bayesian method
  - Has the ability to deal with insufficient data issue, to flexibly select parameter distributions, and to accommodate complicated model structures.

\[
\pi(\alpha, \lambda | q, v) \propto L(\alpha, \lambda | q, v) \pi(\alpha | r_0, m_0) \pi(\lambda | \mu_0, \sigma_0^2)
\]

\[
\alpha \sim \Gamma(r_0, m_0) \quad \lambda \sim N(\mu_0, \sigma_0^2)
\]

- Commonly used priors for Weibull distribution:

- Hierarchical structure
  - Parameters are allowed to vary across freeway sections
  - For the \(k^{\text{th}}\) lane at \(j^{\text{th}}\) section:

\[
\alpha_{kj} \sim \Gamma(r_k, m_k) \quad \lambda_{kj} \sim N(\mu_k, \sigma_k^2)
\]
Bayesian Hierarchical Weibull Model

• Model Assessment
  – Deviance Information Criterion (DIC)
    
    $$DIC = \bar{D}(\theta) + p_D$$

    $\bar{D}(\theta)$: measure of model fitness
    $p_D$: measure of model complexity

• Bayesian Estimation Procedure
  – Bayesian models were estimated via a Markov Chain Monte Carlo (MCMC) algorithm.
  – Two MCMC chains of 30,000 iterations were run (first 10,000 as burn-in).
  – Brooks-Gelman-Rubin (BGR) diagnostic was used to assess the convergence of multiple chains.
### Results & Discussion

- **Modeling Results**
  - Two model structures: standard Weibull vs hierarchical Weibull
  - Two datasets: uncensored vs censored

<table>
<thead>
<tr>
<th>Sci</th>
<th>Median Lane</th>
<th>Center Lane</th>
<th>Shoulder Lane</th>
<th>Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda$</td>
<td>$\lambda$</td>
<td>$\lambda$</td>
<td>$D(\theta)$</td>
</tr>
<tr>
<td></td>
<td>Mean (95% BCI)</td>
<td>Mean (95% BCI)</td>
<td>Mean (95% BCI)</td>
<td>Mean (95% BCI)</td>
</tr>
<tr>
<td>Uncensored Dataset</td>
<td>-60.05 (-66.08, -55.64)</td>
<td>-60.09 (-64.92, -55.05)</td>
<td>-44.68 (-49.15, -40.57)</td>
<td>10132</td>
</tr>
<tr>
<td>Censored Dataset</td>
<td>-65.89 (-71.46, -59.76)</td>
<td>-64.7 (-70.47, -58.37)</td>
<td>-50.45 (-55.33, -45.4)</td>
<td>10543</td>
</tr>
<tr>
<td>Standard Weibull</td>
<td>0.38 (0.02, 1.59)</td>
<td>0.59 (0.02, 3.2)</td>
<td>1.29 (0.02, 5.44)</td>
<td>9610</td>
</tr>
<tr>
<td>Hierarchical Weibull</td>
<td>7.90 (7.33, 8.69)</td>
<td>8.04 (7.37, 8.68)</td>
<td>5.96 (5.41, 6.55)</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>8.64 (7.83, 9.36)</td>
<td>8.61 (7.76, 9.37)</td>
<td>6.69 (6.02, 7.33)</td>
<td>10120</td>
</tr>
<tr>
<td></td>
<td>0.13 (0.07, 0.27)</td>
<td>0.17 (0.08, 0.45)</td>
<td>0.32 (0.11, 0.62)</td>
<td>10549</td>
</tr>
<tr>
<td></td>
<td>10.12 (9.09, 11.02)</td>
<td>10.00 (8.74, 11.06)</td>
<td>11.65 (10.3, 12.7)</td>
<td>9626</td>
</tr>
<tr>
<td></td>
<td>9.33 (8.28, 10.29)</td>
<td>9.75 (8.94, 10.24)</td>
<td>10.15 (9.13, 10.96)</td>
<td>10137</td>
</tr>
</tbody>
</table>
# Results & Discussion

## Model Comparisons

<table>
<thead>
<tr>
<th></th>
<th>Uncensored Dataset</th>
<th>Censored Dataset</th>
<th>* Inconsistent results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median Lane</td>
<td>Center Lane</td>
<td>Shoulder Lane</td>
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<tr>
<td><strong>Standard Weibull</strong></td>
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</tr>
<tr>
<td>25th Percentile</td>
<td>1702</td>
<td>1506</td>
<td>1469</td>
</tr>
<tr>
<td>50th Percentile</td>
<td>1902</td>
<td>1680</td>
<td>1702*</td>
</tr>
<tr>
<td>75th Percentile</td>
<td>2077</td>
<td>1832</td>
<td>1913*</td>
</tr>
<tr>
<td><strong>Hierarchical Weibull</strong></td>
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<td></td>
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</tr>
<tr>
<td>25th Percentile</td>
<td>1692</td>
<td>1434</td>
<td>1395</td>
</tr>
<tr>
<td>50th Percentile</td>
<td>1846</td>
<td>1566</td>
<td>1504</td>
</tr>
<tr>
<td>75th Percentile</td>
<td>1977</td>
<td>1678</td>
<td>1596</td>
</tr>
</tbody>
</table>

The 25th, 50th and 75th Percentiles of the Estimated Capacity Distributions (veh/h/lane)

* Inconsistent results

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![Hierarchical Weibull Non-Censoring](image1)

![Hierarchical Weibull Censoring](image2)
Results & Discussion

- Lane-Specific Capacity Distributions

  - Breakdown Probability
  - Breakdown Hazard

  • Breakdown probability/hazard at certain volume: median lane < center lane < shoulder lane.
  • Breakdown hazards are greater as the volume increases.
  • Cross section-based capacity distribution is close to that of the center lanes, but is significantly different from those of the median and shoulder lanes.
Results & Discussion

• Difference of Capacity Distributions among Freeway Sections

Standard deviations of parameters across sections (i.e. \( sd(\lambda) \) and \( sd(\alpha) \)): median lane<center lane<shoulder lane
Summary & Conclusions

- A method to obtain the **optimal threshold speed** by maximizing the average reduction of efficiency was proposed.
- **Log-rank** and **Wilcoxon** tests were conducted and the results confirmed the heterogeneity of capacity distributions among lanes.
- A **Bayesian hierarchical Weibull model** based on censored capacity data was used to estimate these lane-specific capacity distributions.
  - Estimate breakdown probability for each individual lane
  - Allow parameters to vary across freeway sections
  - Censored data is appropriately treated
  - Bayesian approach is adopted
- Diagnose bottlenecks with **semi-congested** cases and guide vehicles to choose uncongested lanes to reduce breakdown occurrence
Thank You!

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