Calibrating Multilane First-order Traffic Flow Model with Endogenous Representation of Lane-flow Equilibrium

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Lane traffic management in ATM

Active Traffic (and Demand) Management (ATM)
Various menus about lane managements

- HOV and HOT lane
- Keep left (or right) recommendation
- Optic flow for speed control by vection effect
- Ramp metering
- Various speed limit
- Incident management
- Merging channelization
- Hard shoulder opening
- Additional lane etc.

Not well organized and optimized as comprehensive lane management
Model-based decision support system for freeway management

- The needs for the lane-flow management to increase bottleneck capacity, but...

![Diagram showing the relationship between online data collection, online traffic state estimation, traffic state prediction, optimization, and network flow simulation.]

- Data base, knowledge and experiences of controllers
Challenges

Lane-changing is a *microscopic and stochastic maneuver* depending on the surrounding conditions.

- Calibration of the microscopic modeling requires high resolution data (ex. trajectories)
- High cost for data collection

How to overcome?

- A macroscopic modeling of multi-lane traffic including lane-changes
  - Parsimony representation for calibration
  - Reasonable reproducibility of lane-flow phenomena
Objectives

- To develop a macroscopic multilane first-order traffic flow model which endogenously represents lane-flow equilibrium
- To calibrate the parameters in the proposed model on the basis of conventional traffic detectors collected at sag section
MULTILANE FIRST-ORDER TRAFFIC FLOW MODEL
- Model framework
- Lane flow equilibrium and lane changes
- Numerical solution of multi-lane flow

PARAMETER CALIBRATION AT SAG SECTION
- Calibration methods
- Case studies

CONCLUSIONS
- Discussions
- Future works

Outline
Assumptions and limitations

- Fundamental Diagram is defined to each lane
  - Speed is monotonously decreasing as density increases.

- Spontaneous lane changes are considered
  - Not include mandatory lane changes such as towards off ramp or from on ramp.

- Traffic flow consists of homogeneous vehicles
  - Not consider the differences between a passenger car and a large trailer
Multilane first-order traffic flow

- **LWR model**
  - Fundamental Diagram: $V = f(K)$
  - Conservation law: 
    
    $\frac{\partial K}{\partial t} + \frac{\partial (KV)}{\partial x} = 0$

- **Godunov Scheme**
  - $A_{it} = \min \left( S_{ti}, R_{t,i+1}, \left( k_{ji} - K_{t,j+1} \right) \cdot \Delta x \right)$
  - where
    
    $S_{ti} = \begin{cases} 
    K_{ti} \cdot V_{ti} \cdot \Delta t & \text{if } 0 \leq K_{ti} \leq k_{ci} \\
    k_{ci} \cdot v_{ci} \cdot \Delta t & \text{otherwise}
    \end{cases}$

    $R_{t,j+1} = \begin{cases} 
    k_{ci+1} \cdot v_{ci+1} \cdot \Delta t & \text{if } 0 \leq K_{t,j+1} \leq k_{ci+1} \\
    K_{t,j+1} \cdot V_{t,j+1} \cdot \Delta t & \text{otherwise}
    \end{cases}$

Multilane (Laval and Danganzo, 2006)

- Lane specific FD: $V_l = f_l(K_l)$
- Conservation law considering flow balance among lanes:
  
  $\frac{\partial K_l}{\partial t} + \frac{\partial (K_l V_l)}{\partial x} = \Phi_l$
  
  where $\Phi_l = \sum_{l' \neq l} \Phi_{l' \rightarrow l} - \sum_{l' \neq l} \Phi_{l \rightarrow l'}$

- Balancing term
- Come from
- Out to
  
  \[ l - 1 \quad l \quad l + 1 \]
Lane flow distribution

- Characteristic relationship between total density and fraction of lane flow (= lane flow distribution).

- Lane flow distribution is considered as the equilibrium situation, where the costs of using each lane is stochastically balanced.

- Lane changes is represented as the dynamics towards the equilibrium situation.
Motivations for spontaneous lane changes

<Motivation towards the outer lanes>

- To follow keep left (or right) rule

<Motivation towards the inner lanes>

- To overtake slower vehicles to shorten the travel time
Specification of cost function

- Two motivations for spontaneous lane changes
  
  <Motivation towards the outer lanes>
  - To follow keep left (or right) rule
  
  <Motivation towards the inner lanes>
  - To overtake slower vehicles to shorten the travel time

- Cost for a vehicle \( n \) to use lane \( l \) at section \( i \): \( c_{nil}(k_{ilt}) \)

\[
c_{nil}(k_{ilt}) = \alpha_{nil} + \beta_{nil} \cdot \left\{ f_{il}(k_{ilt}) \right\}^{-1} + \varepsilon
\]

A constant value defined to each lane: the cost to break keep left rule
Specifcation of cost function

- Motivations for spontaneous lane changes
  
  **<Motivation towards the outer lanes>**
  - To follow keep left (or right) rule
  
  **<Motivation towards the inner lanes>**
  - To overtake slower vehicles to shorten the travel time

- Cost for a vehicle \( n \) to use lane \( l \) at section \( i \):
  \[
  c_{nil}(k_{ilt}) = \alpha_{nil} + \beta_{nil} \cdot \left\{ f_{il}(k_{ilt}) \right\}^{-1} + \varepsilon
  \]

  A non-negative parameter indicating the sensitivity to travel time

  Inverse of speed
  (= travel time for a unit of distance)

  \( f_{il}(k_{ilt}) : \) Fundamental diagram
Specification of cost function

- Motivations for spontaneous lane changes
  <Motivation towards the outer lanes>
  - To follow keep left (or right) rule
  <Motivation towards the inner lanes>
  - To overtake slower vehicles to shorten the travel time

- Cost for a vehicle \( n \) to use lane \( l \) at section \( i \): \( c_{nil}(k_{ilt}) \)

\[
c_{nil}(k_{ilt}) = \alpha_{nil} + \beta_{nil} \cdot \left\{ f_{il}(k_{ilt}) \right\}^{-1} + \varepsilon
\]

An error term:
Heterogeneity of driver, and
limited recognition
Lane choice probability and lane flow equilibrium

- Let $\epsilon$ follows Weibull distribution with $(0, \theta)$, the probability of a vehicle chooses lane $l$ under the given densities $K (= k_1 + \ldots + k_n)$

$$p_{il}(K_{it}) = \frac{\exp[-\theta \cdot c_{il}(k_{ilt})]}{\sum_k \exp[-\theta \cdot c_{ik}(k_{ikt})]}$$

- At the equilibrium state,

$$p_{il}^*(K_{it}) = \frac{\exp[-\theta \cdot c_{il}(k_{ilt}^*)]}{\sum_k \exp[-\theta \cdot c_{ik}(k_{ikt}^*)]} = \frac{k_{ilt}^*}{K_{it}}$$

$\leftarrow$ Lane choice probability

$\leftarrow$ Lane fraction
SUE in lane choice

Choose a lane for the next cell to reduce the perceived cost

As traffic moves, lane flow approaches Stochastic User Equilibrium (SUE)
Equivalent optimization problem

- SUE is solved by the problem,

\[
\min Z(k) = \sum_{k} \int_{0}^{k_k} c_k(\omega) \, d\omega + \frac{1}{\theta} \sum_{k} k_k \ln \frac{k_k}{K}
\]

subject to

\[K = \sum_{k} k_k\]

\[k_k \geq 0\]

- By applying MSA,

\[
\min Z(y) = \sum_{k} y_k c_k(z_k) + \frac{1}{\theta} \sum_{k} y_k \ln \frac{y_k}{Y}
\]

subject to

\[Y = \sum_{k} y_k\]

\[y_k \geq 0\]

- KKT condition

\[y_k^* = Y \cdot \frac{\exp[-\theta \cdot c_k(z_k)]}{\sum_{j} \exp[-\theta \cdot c_j(z_j)]}\]

* Lane choice probability can be defined by the current lane cost
Numerical treatments of lane flow

- The operation gives the better solution of SUE

Lane density at time step $t$, cell $i$

$$z(t+1, i+1) = z(t, i) + \left(\frac{1}{\tau}\right)\{y^* - z(t, i)\}$$

An adjustment parameter ($\leq 1$)

The number of time step a driver takes to execute a lane change.

- KKT condition

$$y_k^* = Y \cdot \frac{\exp[-\theta \cdot c_k(z_k)]}{\sum_j \exp[-\theta \cdot c_j(z_j)]}$$

- Capacity constrains on the adjacent lanes are considered by IT principle (Laval and Daganzo, 2006)

Change the lanes based on the cost at cell $(t, i)$

Outside
Middle
Median

Approaching SUE
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How to calibrate?

- Step 1: Calibrate parameters of fundamental diagram by Least Square Method using detector data.
  (Estimation parameters: free speed, critical density, critical speed, and jam density)
How to calibrate?

• Step 1: Calibrate parameters of fundamental diagram by Least Square Method using detector data.

• Step 2: Give tentative parameters to the cost function

\[ c_{nil}(k_{ilt}) = \alpha_{nil} + \beta_{nil} \cdot \left\{ f_{il}(k_{ilt}) \right\}^{-1} + \epsilon \]

On outside lane the parameters are fixed as \( \alpha = 0, \beta = 1.0 \)
How to calibrate?

- **Step 1**: Calibrate parameters of fundamental diagram by Least Square Method using detector data.

- **Step 2**: Give tentative parameters to the cost function.
  
  \[ c_{nil}(k_{ilt}) = \alpha_{nil} + \beta_{nil} \cdot \left\{ f_{il}(k_{ilt}) \right\}^{-1} + \varepsilon \]

  On outside lane the parameters are fixed as \( \alpha = 0, \beta = 1.0 \)

- **Step 3**: Lane-flow equilibrium curve is calculated on an imaginary ring road.
• Step 1: Calibrate parameters of fundamental diagram by Least Square Method using detector data.

• Step 2: Give tentative parameters of the cost function.

\[ c_{n|l|t}(k_{i|l|t}) = \alpha_{n|l|t} + \beta_{n|l|t} \cdot \left\{ f_{i|l}(k_{i|l|t}) \right\}^{-1} + \epsilon \]

On outside lane the parameters are fixed as \( \alpha = 0, \beta = 1.0 \)

• Step 3: Lane-flow equilibrium curve is calculated on an imaginary ring road.

• Step 4: Such parameters are found that minimizes the residual error by Quasi-Newton Method.
Study site

- 3 lanes section (for one way), Chugoku expressway in Japan
- 30 days obs. (from Mar, 2010 to Sep, 2010)
- 5 min aggregation for Q and V, makes K

Moving direction

To Osaka

Altitude

-5.0 %  -2.2 %  -4.0 %  -1.7 %  +2.3 %  -2.0 %  -3.0%

Kilopost [km]

25.20 kp  23.12 kp  20.90 kp  20.32 kp

Takaraduka-w TN

Takaraduka-w TN (BN)

Aobadai TN
Calibration results

- Quasi-Newton method gives the convergence solutions.

![Diagram](image_url)

- Altitude
  - Aobadai TN: -2.2%
  - Takaraduka-w TN (BN): -1.7%
  - Moving direction
  - Kilopost [km]:
    - 25.20 kp
    - 23.12 kp
    - 20.90 kp
    - 20.32 kp

![Lane flow distribution](image_url)

- Density [veh/km/3lanes]
  - Outside
  - Middle
  - Median
  - Outside (est)
  - Middle (est)
  - Median (est)
Calibration results

- Quasi-Newton method gives the convergence solutions.
\( \alpha: \) Cost breaking keep-left rule

When traffic density is low, more traffic is likely use on the outside lane, and the middle lane follows.
- $\beta$: Sensitivity to travel time

When traffic density is high, more traffic is likely to use the median lane, and the middle lane follows.
Discussions on the estimated parameters

- $\alpha$: Cost for breaking “keep-left rule”

20.90kp (サグ底地点)，20.32kp (上り勾配)でやや高い値（＝使われにくい）

追越車線の方が高い値（＝低密度時には利用されにくい）

23.13kp (下り勾配)，20.90kp (サグ底)ではやや低い値（＝使われやすい）
Discussions on the estimated parameters

- **β**: Sensitivity to travel time

  - 20.90kp (サグ底) でやや低い値（＝感度が低い）
  - 第2走行車線の方が感度が高い（＝混雑時に他車線へシフトしやすい）
  - 23.13kp（下り勾配）、20.90kp（サグ底）ではやや高い値（＝他車線へ流出しやすい）

![Graph showing estimated parameters](image-url)
Discussions on the estimated parameters

-ボトルネックへ至るまでの車線利用特性

下り区間からサグ底に掛けて相対的に第2走行車線への利用が集中

上り区間で追越車線への利用がシフト
Conclusions

- Multilane first-order traffic flow model, which endogenously represents the lane flow equilibrium
  - Lane flow distribution
  - Traffic dynamics among lanes along characteristic wave
- Calibration for
  - 3 unknown parameters for FD per lane
  - 2 unknown parameters for the cost function for lane choice
  - Good representations of lane-flow equilibrium
Future works

- The adjustment parameter, $\tau$
  
  \[ z(t+1, i+1) = z(t, i) + \frac{1}{\tau} \left( y^* - z(t, i) \right) \]

- Empirical and quantitative analysis about the volume of lane-changes

- Elaborate the multi-lane traffic flow model
  - Mandatory lane changes
  - Multiclass modeling
  - Foreseeing behaviors
  - Disturbance caused by lane-changes, etc.

Thank you for your attention. Any question and comment?