Calibration of Nonlinear Car-Following Laws for Traffic Oscillation Prediction

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Traffic Oscillation

- Stop-and-go traffic – “… a nuisance for motorists throughout the world…” (Zheng et al., 2011)

(Source: http://www.ngsim.fhwa.dot.gov/)  (Adapted from Li et al., 2010)
Trajectory Decomposition

- Vehicle trajectory (or speed, acceleration profiles) exhibit both macroscopic and microscopic characteristics

\[
\hat{x}_l(t) = x_l(t) - \bar{x}_0(0) - \bar{v}t + \sum_{l'=1}^{l} s_{l'}, \forall t,l = 0,1,L \ldots L.
\]

(Li and Ouyang, 2011)
Goal: Predicting Oscillation Propagation

- **Given**
  - Car-following behavior/law
  - Leading vehicle trajectory

- **Determine**
  - Oscillation properties (e.g., amplitude and period) of all following trajectories

Car-following behavior has a significant impact on oscillation.
Linear vs. Nonlinear Car Following Models

Linear Model

Simulation, or Transfer function analysis (e.g., Herman et al., 1958; Chandler et al. 1958)

Field Observation (NGSIM)

Time- (e.g., Neubert et al. 1999; Mauch and Cassidy, 2002) and frequency-domain measurements (Li et al. 2010; 2012; Zheng et al. 2011; Zhao et al. 2014)
Frequency-Domain Measurements

Detrended time-domain data

Frequency spectrum

(Li et al., 2010)
Linear vs. Nonlinear Car Following Models

Linear Model

Nonlinear Model (with physical constraints)

Field Observation (NGSIM)

Simulation, or Transfer function analysis (e.g., Herman et al., 1958; Chandler et al. 1958)

Simulation, or Describing function analysis (e.g., Li and Ouyang 2011; Li et al. 2012)

Time- (e.g., Neubert et al. 1999; Mauch and Cassidy, 2002) and frequency-domain measurements (Li et al. 2010; 2012; Zheng et al. 2011; Zhao et al. 2014)
A Class of Nonlinear Car-Following Laws

• Car-following law structure

\[ \hat{x}_l = G_l \cdot F_l(\hat{x}_{l-1} - \hat{x}_l), \forall l \]

- \( F_l \): Nonlinear functional (e.g., desired speed as a function of spacing)
  - Non-decreasing
  - Lipschitz continuous
  - Bounded
- \( G_l \): Linear operator (e.g., driver reaction, estimation, etc)
  - Low-frequency-pass (Boashash, 2003)
  - E.g., integral, differential and time-shift

Examples: All linear models, Newell’s nonlinear model (1961), optimal velocity (Bando et al., 1995; 1998), among many others

(Li and Ouyang, 2011)
Car-Following Law Examples

\[ \hat{x}_l = G_l \cdot F_l (\hat{x}_{l-1} - \hat{x}_l) \]

- **Examples 1**: Newell’s Model (1961)
  - \( F_l \): triangular fundamental diagram
  - \( G_l \): time-lag (reaction time)

- **Example 2**: Optimal velocity model (Bando et al., 1995; 1998)
  - \( F_l \): hyperbolic tangent function
  - \( G_l \): satisfies a first-order differential equation with time-lag
Analytical Prediction

- Describing function analysis (Slotine, 1990; Mees, 1981) + feedback loop

\[ \hat{x}_l = G_l \cdot F_l(\hat{x}_{l-1} - \hat{x}_l) \]

(Li and Ouyang, 2011; Li et al. 2012)
Example of Oscillation Prediction

- Newell’s Model (1961)

\[
\frac{dx_i(t)}{dt} = \text{mid} \left( 0, \lambda_i [x_{i-1}(t - \tau) - x_i(t - \tau) - s_i^*], v^* \right)
\]

- Oscillation propagation

![Graphs showing oscillation propagation for different vehicle inputs and velocities](image-url)

\( s = x_{i-1}(t - \tau) - x_i(t - \tau) \)
Need for Car-Following Model Calibration

**Field data**
- Measurement

**Car-following model calibration**
- Linear car-following law
  - Transfer function analysis
- Nonlinear car-following law
  - Describing function analysis

**Validation**
- Analytical prediction of oscillation properties, e.g. amplitude growth
  - Computer simulation
  - Simulated oscillation properties
  - Analytical prediction of oscillation properties
  - Observed oscillation properties
A straightforward idea is to fit a Tobit model; i.e., for any reaction time \( \tau \); assume

\[
v_i(t) = \begin{cases} 
0 & \text{if } \theta_i(t) < 0, \\
v_{\max} + \varepsilon_i & \text{if } s_i(t - \tau_i) \geq s_i^{\max}, \\
\theta_i(t) + \varepsilon_i & \text{otherwise},
\end{cases}
\]

where \( \theta_i(t) \) is the linear branch, and \( \varepsilon_i \sim N(0, \sigma) \), i.i.d.

Estimate five independent parameters \(<k, \tau, \omega, v_{\max}, \sigma>\) by maximizing the following likelihood function:

\[
L = \prod_{t \in T_i; \hat{v}_i(t) < 0} \Phi \left( \frac{\hat{v}_i(t)}{\sigma} \right) \cdot \prod_{t \in T_i; s_i(t, \tau_i) \geq s_i^{\max}} \left[ \frac{1}{\sigma} \phi \left( \frac{v_i(t) - v_{\max}^i(t)}{\sigma} \right) \right] \cdot \prod_{t \in T_i; (\hat{v}_i(t) \geq 0) \& (s_i(t, \tau_i) < s_i^{\max})} \left[ \frac{1}{\sigma} \phi \left( \frac{v_i(t) - \hat{v}_i(t)}{\sigma} \right) \right]
\]
Issues of the Simple Tobit Model

- Time-domain
  - Velocity-spacing diagram depends highly (and nonlinearly) on reaction time
  - Continuity and autocorrelation among these scatter points over time are not considered
- Frequency-domain
  - No explicit consideration of oscillation properties
- Two types of expected prediction errors
Penalty-based Calibration

• For each pair of trajectories (or all pairs), estimate parameters by solving the following constrained optimization problem

\[
\begin{align*}
\text{max} & \quad \log L \\
\text{s.t.} & \quad \text{physical constraints (e.g., } \tau \geq 0) \\
& \quad \text{time-domain MSE } \leq \text{bound}_T \\
& \quad \text{frequency-domain error } \leq \text{bound}_F
\end{align*}
\]

• Modified penalty method

\[
\begin{align*}
\text{max} & \quad \log L - \lambda_T \left( \text{time-domain MSE } - \text{bound}_T \right) - \lambda_F \left( \text{frequency-domain error } - \text{bound}_F \right) \\
\text{s.t.} & \quad \text{physical constraints (e.g., } \tau \geq 0) 
\end{align*}
\]

where \( \lambda_T \) and \( \lambda_F \) are multipliers that are adjusted via iterations.

• For each iteration, solve the above problem using meta-heuristic (e.g., simulated annealing)
Field Data: NGSIM Trajectories

- **Trajectory Data**
  - US 101 (Hollywood Freeway), Los Angeles, California
  - Between 7:50 to 8:35 am June 15th, 2005

- **Frequency-domain**
  - Amplitude/frequency measurement
  - Prediction (describing function analysis)

- **Time-domain**
  - Simulation and trajectory reconstruction
Platoon #1 (Trajectories 81-93)

- Estimated parameters per driver pair
- Sample curve-fitting plots

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>$k$</th>
<th>$\tau$ (sec)</th>
<th>$\omega$ (ft/s)</th>
<th>$v_{\text{max}}$ (ft/s)</th>
<th>$\sigma$</th>
<th>$e^F$</th>
<th>$e^T$</th>
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</table>
Reproduction

- Time-domain: trajectory
- Frequency domain: amplitude
Platoon #2 (Trajectories 311-320)

- Estimated parameters
- Time- and frequency-domain reproduction

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>$k$</th>
<th>$\tau$ (sec)</th>
<th>$\omega$ (ft/s)</th>
<th>$v_{\text{max}}$ (ft/s)</th>
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<td>7.44</td>
<td>1.17 e-4</td>
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</table>
• If we ignore either reproduction feedback…

Time-domain feedback only

Frequency-domain error only

81st to 93rd Trajectories

Observation
Reproduction

Observation
Reproduction
Prevailing “String Instability” Issues

In congested traffic, fluctuations in flow / speed tend to grow upstream (stop-and-go) (Empirical data: Mauch and Cassidy, 2002)

In supply chains, order variability tends to increase as orders move up the chain. (Empirical data: Lee et al., 1997)
Prediction under Nonlinear Dynamics

Traffic oscillation growth

Supply chain order variability growth

(Li and Ouyang, 2011)  (Wang et al., 2014)
Summary

Describing function analysis

Observed oscillation properties

Measurement

Nonlinear car-following law

Computer simulation

Simulated oscillation properties

Analytical prediction of oscillation properties

Describing function analysis

Validation

Verification

Field data

Car-following model calibration

Measurement

Observed oscillation properties

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Thank You

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Future Work

• Other platoons, datasets?
  – Field observations?
• Other car-following law models (e.g., OV model)?
  – DFA assumptions (bounded etc.)
  – Calibration assumptions
• Other calibration objectives/methods (MLE vs. GMM)?
• Other computation/optimization algorithms?
• Nonparametric car-following laws?
• Mitigation strategies?
• Other contexts?